Matter–wave emission in optical lattices: Single particle and collective effects

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We introduce a simple set–up corresponding to the matter-wave analogue of impurity atoms embedded in an infinite photonic crystal and interacting with the radiation field. Atoms in a given internal level are trapped in an optical lattice, and play the role of the impurities. Atoms in an untrapped level play the role of the radiation field. The interaction is mediated by means of lasers that couple those levels. By tuning the lasers parameters, it is possible to drive the system through different regimes, and observe phenomena like matter wave superradiance, non-Markovian atom emission, and the appearance of bound atomic states.

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Recent progress in atomic physics has allowed experimentalists to trap atoms in optical potentials at very low temperatures. This has led to the observation of several interesting phenomena in which atom–atom interactions play a predominant role. With atoms loaded in optical lattices it is nowadays possible, for example, to reach the strong correlation regime where quantum phase transitions between superfluid and insulator phases [1, 2], Tonks–Girardeau gases [3], or even entanglement between neighboring atoms can be observed. Those experiments have triggered a large amount of theoretical work proposing and analyzing new experiments where intriguing condensed matter behavior could be observed.

In this work we show that with the same systems it is possible to observe a broad spectrum of different phenomena usually connected to light–matter interactions (see also [4, 5, 6] for related setups). In our setup, the role of matter is played by the absence/presence of one atom in the ground state of an optical potential, whereas the role of light is played by weakly–interacting atoms in a different internal state which is not affected by the optical potential. The coupling between those two systems is induced by Raman lasers, which connect the two internal states of each atom (see Fig. 1). As we will show, the Hamiltonian that describes this situation is very similar to that describing the interaction between two–level atoms and the electromagnetic field within a photonic crystal (PC). By changing the laser and optical trapping parameters it is possible to drive the system to different regimes where a rich variety of phenomena can be observed. These include the spontaneous polarization of the system predicted by the mean field theory [7], collective effects in the emission of atoms from the lattice [8, 9**?**], and the formation of a bound trapped–untrapped atom state, analogous to the atom–photon bound state that appear when atoms within a photonic crystal emit photons within the gap region [10, 11, 12, 13]. Moreover, it is possible to reach a regime in which weakly confined atoms drive atom–atom interactions between strongly confined ones, giving rise to effective Coulomb-like interactions between them.

We consider N cold atoms with a ground state hyperfine level $|a\rangle$ and frequency ω_a^0 that is trapped by an optical lattice with M sites and lattice period d_0. The motion of the atoms is restricted to the lowest Bloch band in the collisional blockade regime, where either one or no atom occupy each potential well of the lattice, which we will approximate by a harmonic oscillator of frequency ω_0. Then, we can replace the creation operator at each site by $\sigma_{\mathbf{j}}^+ = |1\rangle_{\mathbf{j}}\langle 0|$, which describes transitions from the Fock state $|0\rangle_{\mathbf{j}}$ with no atoms at site $\mathbf{j}$, to a state $|1\rangle_{\mathbf{j}}$ with one atom [4]. The atoms have an additional internal level, $|b\rangle$, that is not affected by the lattice potential (what can be achieved with state dependent potentials) and has a frequency ω_b. We introduce the field operator $\psi_b^\dagger(\mathbf{r}) = (1/\sqrt{V})\sum_{\mathbf{k}} b_{\mathbf{k}}^\dagger e^{i\mathbf{k}\cdot\mathbf{r}}$, where V is the quantization volume, and $b_{\mathbf{k}}^\dagger$ is the creation operator of an atom in $|b\rangle$ with momentum $\mathbf{k}$.

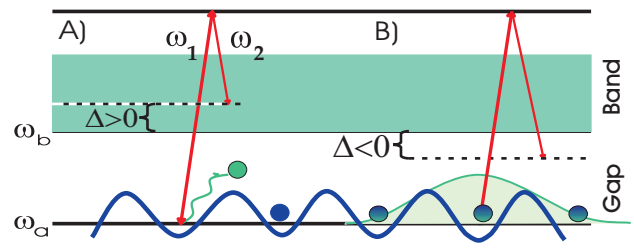

FIG. 1: (Color online) Schematic diagram: Lasers produce two-photon Raman transitions to an untrapped state. Fig A: For laser detunings $\Delta > 0$, transitions are into a *band* of non–trapped states. Fig B: $\Delta < 0$, transitions are into the *gap* region, and a trapped-untrapped atom bound state is formed.

In a similar way as in an atom laser setup [14], two lasers are then used to induce two–photon Raman transitions between $|a\rangle$ and $|b\rangle$. The lasers have a two–photon Rabi frequency Ω, and their frequencies and momentum differences are $\omega_L = \omega_1 - \omega_2$ and $\mathbf{k}_L = \mathbf{k}_1 - \mathbf{k}_2$ respectively. When tuning them close to a two photon resonance and far from single photon resonances, an effective Hamiltonian is obtained which in the interaction picture

can be written as ($\hbar = 1$)

$$H_{\rm int} \;=\; \sum_{\mathbf{j}}\sum_{\mathbf{k}} g_k \left(b^\dagger_{\mathbf{k}} \sigma_{\mathbf{j}} e^{i\Delta_{\mathbf{k}} t - i(\mathbf{k}-\mathbf{k}_L)\cdot \mathbf{r}_{\mathbf{j}}} + h.c. \right). \quad (1)$$

Here $\mathbf{r}_{\mathbf{j}}$ denotes the positions in the lattice, and $\Delta_{\mathbf{k}} = k^2/2m - \Delta$, with $\Delta = \omega_L - (\omega_b - \omega_a)$ the laser detuning (and $\omega_a = \omega_a^0 + \omega_0/2$). The coupling constants are $g_k = \Omega e^{-X_0^2 k^2/2}(8\pi^{3/2} X_0^3/V)^{1/2}$, where $X_0 = (1/2m\omega_0)^{1/2}$ is the size of the wave function at each site.

The similarity of Hamiltonian (1) with that describing the interaction of atoms with the electromagnetic field is apparent. The dispersion relation of the atomic bath is contained in Δ_k, and resembles that of the radiation field in a three dimensional and infinite PC near the band-edge [7, 11, 13]. Furthermore, in our set–up one can easily control several external parameters: Ω, which determines the coupling strength, Δ, which determines the resonance conditions, the number of atoms N and of sites M, the lasers wavevectors $\mathbf{k}_L$, and the dimension of the trap and the lattice. Thus, we expect to observe a rich variety of phenomena with our system, ranging from some well–known from the field of Quantum Optics, to other which are difficult to access in that field. We will start out with a single excitation ($N = M = 1$), and then consider collective effects ($M > 1$), for the different regimes dictated by the control parameters Ω and Δ.

An atom within a single trap constitutes the simplest setup but it still gives a very good insight into the problem. The wave function of the system has the form $|\Psi(t)\rangle = A(t)|1,\{0\}\rangle + \sum_{\mathbf{k}} B_{\mathbf{k}}(t)|0,1_{\mathbf{k}}\rangle$, where $|1,\{0\}\rangle$ describes the atom in the trapped state and no free atom present, and $|0,1_{\mathbf{k}}\rangle$ represents no atom in the trapped state and a single untrapped atom in the mode $\mathbf{k}$. Using the Schrödinger equation we have $\dot{A}(t) = -\int_0^t d\tau G(t-\tau)A(\tau)$, where

$$G(t) = \sum_{\mathbf{k}} g_k^2 e^{-i\Delta_{\mathbf{k}} t} = \Omega^2 \frac{e^{i(\Delta t - \arctan[\omega_0 t])}}{\nu_t |\nu_t|^2}, \quad (2)$$

with $\nu_t = 2\sqrt{1 + i\omega_0 t}$, is the correlation function of the environment. An analytical solution can be obtained by assuming that trapped atoms are strongly confined so that $\omega_0 \gg \Omega, \Delta$. For a $3D$ bosonic field, this leads to $G_\infty(t) = -\alpha e^{i(\Delta t + \pi/4)}/t^{3/2}$, which is singular at the origin, but describes correctly times $t \gg 1/\omega_0$. Except for the value of $\alpha = \Omega^2/\omega_0^{3/2}$, $G_\infty(t)$ is identical to the correlation function of the radiation field within an anisotropic PC, as described in [12]. In the same way, a $1D$ environment (produced by trapping the atoms in $|b\rangle$ in a $1D$ harmonic trap), gives rise to a correlation function similar to the one appearing for the radiation field in isotropic PCs. Using the Laplace transform method, we get [12]

$$A(t) = c_1 e^{i(r_1^2 + \Delta)t} + I(\alpha, \Delta, t), \quad (3)$$

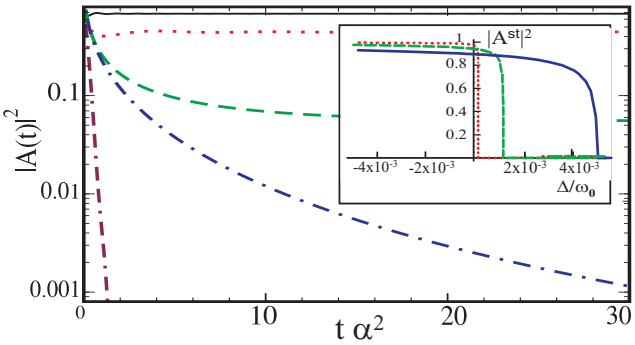

FIG. 2: Evolution of the atomic population $|A(t)|^2$ in logarithmic scale for different detunings. Solid, dotted, dashed, dot-dashed and dot-dot-dashed lines correspond respectively to $\Delta/\alpha^2 = -8, -1, -0.2, 0.2, 8$ ($\omega_0 = \infty$), where one can recognize the regimes explained in the text. Inset [17]: Steady state population $|A^{st}|^2 = |A(\infty)|^2$. Solid, dashed and dotted lines correspond respectively to $\Omega/\omega_0 = 0.05, 0.025, 0.01$.

with $I(\alpha, \Delta, t) = (\alpha e^{i\pi/4}/\pi)\int_0^\infty dx \frac{\sqrt{x} e^{(-x+i\Delta)t}}{(-x+i\Delta)^2 + i\alpha^2 x}$. Defining $r_\pm = -(\alpha/2) \pm \sqrt{(\alpha/2)^2 - \Delta}$ we have: (i) If $\alpha^2/2 > \Delta > 0$, then $c_1 = 0$. (ii) If $\Delta > \alpha^2/2$, then $r_1 = r_-$ and $c_1 = \frac{2r_-}{r_- - r_+}$. (iii) If $\Delta < 0$, $r_1 = r_+$ and $c_1 = \frac{2r_+}{r_+ - r_-}$. Depending on the parameters, we can have very different behaviors. For $\Delta > 0$ there is no trapped atom left in steady state, whereas for $\Delta < 0$ this is not the case (Fig. 2). Thus, there is a quantum phase transition at $\Delta = 0$ analog to that found in the spin–boson model [15]. Moreover, for $\Delta < 0$, the emitted atomic field is in the form of evanescent modes exponentially localized around the trapped atom, what leads to the trapped atom–untrapped atom bound state. This follows from the probability of finding a radiated particle at position $\mathbf{r}$ at a long time t, $|\Psi_b(\mathbf{r},t)|^2 = (c_1 \Omega m X_0^3/2\pi r)^2 e^{-{\rm Im}[k_0^e]r} e^{-{\rm Im}[r_1^2]t}$, where $k_0^e = \sqrt{2m(\Delta - r_1^2)}$ is imaginary if $\Delta < 0$. For $\Delta > 0$ we have two different regimes: for $\Delta \gg \alpha^2$ we just have an exponential rate $\Gamma_0 = 2\Omega^2\sqrt{\pi\Delta/\omega_0^3}$, whereas in the opposite limit the evolution does not follow such a law. The first corresponds to the Markovian regime, where the correlation time $\tau_c \simeq \Delta^{-1}$ of the environment (here untrapped atoms) is shorter than the typical evolution time of the trapped atom (Γ_0^{-1}) [17].

We now study the dynamics of atoms in a lattice with M sites, choosing $\omega_0 \gg \Omega, \Delta$. Guided by the previous analysis, we will consider the regimes $\Delta > 0$ (Markovian and non–Markovian), as well as $\Delta < 0$.

In the limit where $\Gamma_{\rm coll}\tau_c \ll 1$, where $\Gamma_{\rm coll}$ gives the typical evolution time of the trapped atoms, we can analyze the problem under the Born–Markov approximation. The dynamics of the atoms in the lattice is dictated by the quantities

$$\Gamma_{|\mathbf{i}-\mathbf{j}|} = \int_0^\infty d\tau G_{\mathbf{i}-\mathbf{j}}(\tau) = i|\Gamma_0|\xi \frac{e^{-\nu|\mathbf{i}-\mathbf{j}|/\xi + i\mathbf{r}_{\mathbf{i}-\mathbf{j}}\cdot\mathbf{k}_L}}{|\mathbf{i}-\mathbf{j}|}, \quad (4)$$

for $\mathbf{i} \neq \mathbf{j}$. Here, $\nu = i, 1$ for $\Delta > 0$ and $\Delta < 0$, respectively, and the correlation function $G_{\mathbf{i}-\mathbf{j}}(t) = \sum_{\mathbf{k}} g_k^2 e^{i\mathbf{r}_{\mathbf{i}-\mathbf{j}}\cdot(\mathbf{k}-\mathbf{k}_L)-i\Delta_k t}$ is now

$$G_{\mathbf{i}-\mathbf{j}}(t) = G(t) e^{i r^2_{\mathbf{i}-\mathbf{j}}/(4X_0^2\nu_t^2)} \tag{5}$$

with $G(t)$ given by (2). Similar to the radiative case, the coefficients $\Gamma_{|\mathbf{i}-\mathbf{j}|}$ describe the dipolar interactions between the sites $\mathbf{i}$ and $\mathbf{j}$. The quantity $\xi = 1/(|k_0|d_0)$, with $k_0 = \sqrt{2m\Delta}$ [16], quantifies the range of the interactions which, according to (4), has a Yukawa form.

For $\Delta > 0$, the situation under study resembles that of a set of M atoms in a lattice of constant d_0 interacting with the electromagnetic field with resonant wave–vector k_0, and where N corresponds to the number of excited atoms. Thus, phenomena like multiple–scattering, reabsorption, or superradiance should be expected. In fact, apart from ξ we can also define the analogous of the optical depth, which for a cubic lattice is $\chi = M^{1/3}\xi^2$. Depending on the values of those dimensionless parameters we can predict different phenomena.

Let us first consider the case of one atom $N = 1$, symmetrically distributed through a lattice of M sites. We will assume that laser directions are chosen such that $|\mathbf{k}_L| = |k_0|$. The state at time t can be expressed as $|\Psi(t)\rangle = (1/\sqrt{M})\sum_{\mathbf{i}} A_{\mathbf{i}}(t)|1_{\mathbf{i}}, \{0\}\rangle + \sum_{\mathbf{k}} B_{\mathbf{k}}(t)|0, 1_{\mathbf{k}}\rangle$, where $A_{\mathbf{i}}$ represents the amplitude at site $\mathbf{i}$. Following similar lines as in the single site example we can write $\dot{A}_{\mathbf{i}}(t) = \sum_{\mathbf{j}} \Gamma_{|\mathbf{i}-\mathbf{j}|} A_{\mathbf{j}}(t)$, with rates given by (4). An analytical solution can be obtained by considering a large system ($M^{1/3} \gg 1$) such that boundary effects are neglected, and periodic boundary conditions can be assumed. In that situation, atoms in $|a\rangle$ remain in the completely symmetric state with an amplitude $A_{\rm coll}(t) = (1/\sqrt{M})\sum_{\mathbf{i}} A_{\mathbf{i}}(t)$ that decays with a collective rate $\Gamma_{\rm coll} = \sum_{\mathbf{n}} \Gamma_{|\mathbf{n}|}$. Then, the system reaches several regimes in which collective effects play an important role [18]: (a) If $\xi < 1$ and $\chi \gg 1$, reabsorption occurs, and the decay rate is renormalized to $\Gamma_{\rm coll} \sim \chi\Gamma_0$. (b) If $M^{1/3} > \xi > 1$, the dipolar interactions couple near neighbors, and $\chi > 1$ independent of the size of the system. The rate $\Gamma_{\rm coll}$ scales in the same way as in (a). Finally, case (c) corresponds to $\xi \gg M^{1/3}$, a situation in which every site is connected through dipole interactions to all other sites, and $\Gamma_{\rm coll} = M\Gamma_0$.

We now consider N atoms within M sites. The above described regimes still give a valid picture for this situation. In particular, we focus on the collective limit (b), where $\xi > 1$. In addition, from here on, we will consider laser directions to be such that $|\mathbf{k}_L|d_0 M^{1/3} \ll 1$. When all the atoms are initially in the lattice, we get

$$\begin{aligned} \frac{d\langle\sigma_{\mathbf{i}}^3\rangle}{dt} &= -4{\rm Re}[\sum_{\mathbf{j}} \Gamma_{|\mathbf{j}-\mathbf{i}|}\langle\sigma_{\mathbf{i}}^+\sigma_{\mathbf{j}}\rangle] \\ \frac{d\langle\sigma_{\mathbf{i}}^+\sigma_{\mathbf{j}}\rangle}{dt} &= \sum_{\mathbf{l}} \Gamma^*_{|\mathbf{l}-\mathbf{i}|}\langle\sigma_{\mathbf{l}}^+\sigma_{\mathbf{i}}^3\sigma_{\mathbf{j}}\rangle + \Gamma_{|\mathbf{l}-\mathbf{j}|}\langle\sigma_{\mathbf{i}}^+\sigma_{\mathbf{j}}^3\sigma_{\mathbf{l}}\rangle, \end{aligned} \tag{6}$$

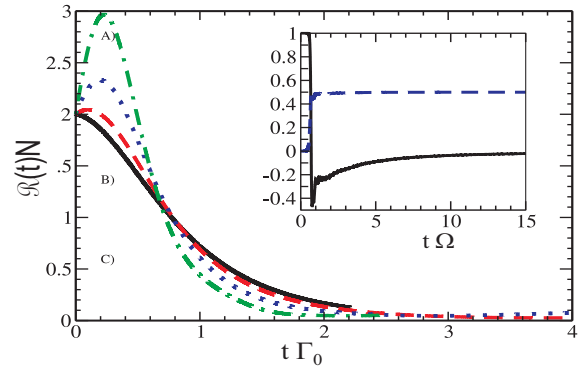

FIG. 3: Rate of atomic emission from a $1D$ lattice with $N = M = 100$ atoms. Solid, dashed, dotted and dot-dashed curves correspond to $\xi = 0.9, 1.25, 2, 3.33$ respectively. Inset [17]: Spontaneous symmetry breaking and non–zero steady state population. Solid and dashed curves represent $z(t)$ and $y(t)$ respectively, evolving according to (8), for $N = M = 10^3$ atoms, $\Delta = 0$, and $\omega_0 = 50$.

with rates given by (4). Here, $\sigma_{\mathbf{i}}^3 = 2\sigma_{\mathbf{i}}^+\sigma_{\mathbf{i}} - 1$, and all the operators are evaluated a time t. Let us first analyze the atomic emission that occurs for positive detuning $\Delta > 0$. We focus on the rate of emission of atoms in all directions, which is given by $\mathcal{R}(t) \approx -\sum_{\mathbf{j}} d\langle\sigma_{\mathbf{j}}^3\rangle/dt$, for different values of ξ. If sites evolve independently, $\mathcal{R}(t)$ decays exponentially. However, when $\xi > 1$ and collective effects are present, $\mathcal{R}(t)$ does no longer decay exponentially and, furthermore, it presents positive slopes at initial times. This is shown in Fig. 3 for a $1D$ lattice, where it is observed that collective effects occur for $\xi > 1$. This result is obtained with (6) by using the semiclassical decoupling $\langle\sigma_{\mathbf{l}}^+\sigma_{\mathbf{i}}^3\sigma_{\mathbf{j}}\rangle = \langle\sigma_{\mathbf{i}}^3\rangle\langle\sigma_{\mathbf{l}}^+\sigma_{\mathbf{j}}\rangle$, that is based on neglecting atomic quantum fluctuations [9]. Nevertheless, the change of sign in the slope can be obtained analytically by differentiating Eq. (6) at $t = 0$ without the use of any approximation.

If we now consider negative detuning, $\Delta < 0$, the rates (4) are purely imaginary, and the system has an effective Hamiltonian

$$H_{\rm eff}^{\Delta<0} = \sum_{\mathbf{i},\mathbf{j}} J_{|\mathbf{i}-\mathbf{j}|}\sigma_{\mathbf{i}}^\dagger\sigma_{\mathbf{j}}^-, \tag{7}$$

where $J_{|\mathbf{i}-\mathbf{j}|} = i\Gamma_{|\mathbf{i}-\mathbf{j}|}$ is a real and negative quantity, so that (7) is Hermitian and describes a coherent spin–spin interaction of ferromagnetic type. This interaction may have interesting applications in the field of quantum simulation. Furthermore, for $\xi \gg 1$ it gives a Coulomb–like interaction very difficult to obtain with other techniques.

We now concentrate in the non–Markovian limit, where the system also becomes strongly interacting. We will consider that all the atoms are initially in the lattice (i.e., $N = M$), and the limit $M \gg 1$. We use the mean field or Hartree approximation [22]. Then, the evolution of $y(t) = \sum_{\mathbf{j}}\langle\sigma_{\mathbf{j}}^-(t)\rangle/M$ and $z(t) = \sum_{\mathbf{j}}\langle\sigma_{\mathbf{j}}^3(t)\rangle/M$ can be

written as

$$\frac{dy(t)}{dt} = M\int_0^t d\tau G_{\rm coll}(t-\tau)y(\tau)z(t);$$
$$\frac{dz(t)}{dt} = -4M{\rm Re}\left[\int_0^t d\tau G_{\rm coll}(t-\tau)y^*(\tau)y(t)\right]. \quad (8)$$

The function $G_{\rm coll}(t)=\sum_{\mathbf{n}} G_{\mathbf{n}}(t)$, with $G_{\mathbf{n}}(t)$ defined in (5). Due to the non–Markovian structure of the equations, the mean field approximation here considered predicts that the trapped atoms acquire a macroscopic polarization in the steady state. This is shown in Fig. (3) for $\Delta=0$, where we have considered an initial infinitesimal polarization, $y(0)=10^{-6}$ (though the steady state does not depend on the choice of $y(0)$), and $z(0)=1$. The spontaneous polarization of the system, previously described in [7] for atoms in PCs, is similar to the spontaneous symmetry breaking described in the semiclassical theory of the laser ([22] and references therein). Fig. (3) also shows that the non–Markovian effects lead to a non–zero steady state population, i.e. $z^{st}\neq -1$.

Most of the phenomena described here may be observed with state of the art experimental setups using state–dependent potentials [1] in the Mott insulator regime [2] for the lattice atoms and choosing $\Omega \ll \omega_0$ to avoid the occupation of other bands. The simplest regime corresponds to the a single excitation ($N=M=1$), where decay, non–Markovian effects, as well as the phase transition occurring at $\Delta=0$ can be observed. Note that it is not required to have a single atom in the whole lattice, as long as the atoms do not interact with each others (i.e. $\xi \ll 1$ and initially localized), so that one could easily monitor the decay as a function of time by simply measuring how many atoms remain in $|a\rangle$. The presence of the bound state of the untrapped atoms should be visible in the momentum distribution after free expansion. By preparing a few atoms each of them coherently distributed among M sites in disjoint regions, several copies of the setup consisting in $N=1$ atom within M sites could be realized. Hence, we would observe collective effects in the decay time for $\chi \geq 1$. For an initial Mott insulator state ($N=M$) it should also be possible to observe superradiant effects by looking at the slope of the decay rate for short times, and for $\Delta<0$ the nearest–neighbor interaction induced via virtual transitions to the untrapped state, as follows from Eq. (7). All those phenomena require $\xi \geq 1$, i.e. $|\Delta| \leq 1/(2md_0^2)$, as well as the Markovian limit, $\Gamma_{\rm coll} \leq |\Delta|$. Observing superradiance for long times (i.e. the whole shape of Fig. 3) may be limited by decoherence effects caused by random magnetic fields which shift $|a\rangle$ and $|b\rangle$ differently. Furthermore, to observe coherent interactions in Eq. (7) beyond nearest neighbors requires $\xi \gg 1$, which may also be compromised by decoherence effects. A possible way around this is to use lighter atomic species, like Li, where those conditions are relaxed. On the other hand, the collecive non–Markovian effects related to the spontaneous polarization should be also easy to observe by choosing $\Delta \simeq 0$.

The proposed set–up may be advantageous to observe some phenomena with respect to atoms interacting with light in a PC. First, it is easily tunable; second, the detection techniques developed for atoms in optical lattices [23] may allow to measure features, like the analogue to the photon-atom bound state, that are difficult to measure in PCs; third, the optical lattice is a nearly perfect periodic potential. In addition, other interesting phenomena could be explored with the present set–up. For example, for $\chi \gg 1$ light–matter interface schemes can be used to control the emission direction of the atoms, or to map the state of trapped atoms into that of untrapped atoms [24]. Besides that, if $|b\rangle$ is affected by a trap that is wider than that of $|a\rangle$, other types of interactions, like the one described by the Jaynes Cummings or the so-called Tavis Cummings model, can be implemented. Finally, the present system may be tuned to explore other regimes which have never been considered in quantum optics since they could not be reached there, like for example the ones in which the initial state of the atoms in the lattice is a superfluid, or a Tonks gas.

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[1] D. Jaksch *et al.*, Phys. Rev. Lett. **81**, 3108 (1998); O. Mandel *et al.*, Nature **425**, 937 (2003).
[2] M. Greiner *et al.*, Nature **415**, 39 (2002); S. Fölling *et al.*, Nature **434**, 481 (2005).
[3] B. Paredes *et al.*, Nature **424**, 277 (2004).
[4] A. Recati *et al.*, Phys. Rev. Lett. 94, 040404 (2005).
[5] Peter P. Orth *et al.*, arXiv:0711.2309 (2007).
[6] K.Molmer, Phys. Rev. Lett. **90**, 110403 (2003).
[7] S. John and T. Quang, Phys. Rev. Lett. **74**, 3419 (1995).
[8] R. H. Dicke, Phys. Rev. **93**, 99 (1954).
[9] A.V. Andreev, V.I. Emel'Yanov, and Yu A. Il'inskii, Cooperative effects in optics, superradiance and phase transitions, Malvern Physics series (1993).
[10] S. John, Phys. Rev. Lett. **58**, 2486 (1987); E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).
[11] S. John and T. Quang, Phys. Rev. A **50**, 1764 (1994).
[12] M. Woldeyohannes and S. John, J. Opt. B: Quantum Semiclass. Opt. **5** (2003), R43-R82.
[13] I. de Vega, D. Alonso and P. Gaspard, Phys. Rev. A **71**, 023812 (2005).
[14] G.M. Moy and C.M. Savage, Phys. Rev. A **56**, R1087 (1997).
[15] A.J. Leggett *et al.*, Rev. Mod. Phys. **59**, 1 (1987).
[16] Note that $k_0^e \approx k_0$, within the weak coupling limit.

[17] For finite ω_0, $\Gamma_0 = \int_0^\infty dt G(t) = \Omega^2(-2i/\omega_0 + 2\sqrt{\pi\Delta/\omega_0^3}e^{-\Delta/\omega_0}(1 + \mathrm{Erf}[i\sqrt{\Delta/\omega_0}]))$, where the first term is the analogue to the Lamb shift. In this case, the structure of $A(t)$ is similar to (3), but the phase transition above described is shifted to $\Delta = 2\Omega^2/\omega_0$ (see inset Fig. 2).

[18] D. Porras and J.I.Cirac, in preparation.

[19] I. H. Deutsch, P.S. Jessen, Phys. Rev. A **57**, 1972 (1998).

[20] G.K. Brennen *et al.* Phys. Rev. Lett. **82**, 1060 (1999).

[21] D. Jacks *et al.*, Phys. Rev. Lett. **82**, 1975 (1999).

[22] H.P. Breuer, and F. Petruccione, The theory of open quantum systems, Oxford Univ. Press (2002).

[23] S. Fölling *et al.*, Nature **434**, 481-484 (2005).

[24] K. Hammerer *et al.* Phys. Rev. A **70**, 044304 (2004).