Preparation of distilled and purified continuous variable entangled states

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The distribution of entangled quantum states of light over long distances is a major challenge in the field of quantum information. Optical losses, phase diffusion and mixing with thermal states lead to decoherence and destroy the nonclassical states after some finite transmission line length. Quantum repeater protocols[1,2] combining quantum memory[3], entanglement distillation[4,5] and entanglement swapping[6] were proposed to overcome this problem. Here we report on the first experimental demonstration of entanglement distillation in the continuous variable regime[7,8,9]. Entangled squeezed states were first disturbed by random phase fluctuations and then distilled and purified using interference on beam splitters and homodyne detection. Measurements of covariance matrices clearly indicated a regained strength of entanglement and purity of the distilled state. Contrasting previous demonstrations in the complementary discrete variable regime[10,11], our scheme achieved the actual preparation of the distilled states, which might therefore be used to improve the quality of down-stream applications such as quantum teleportation[12].

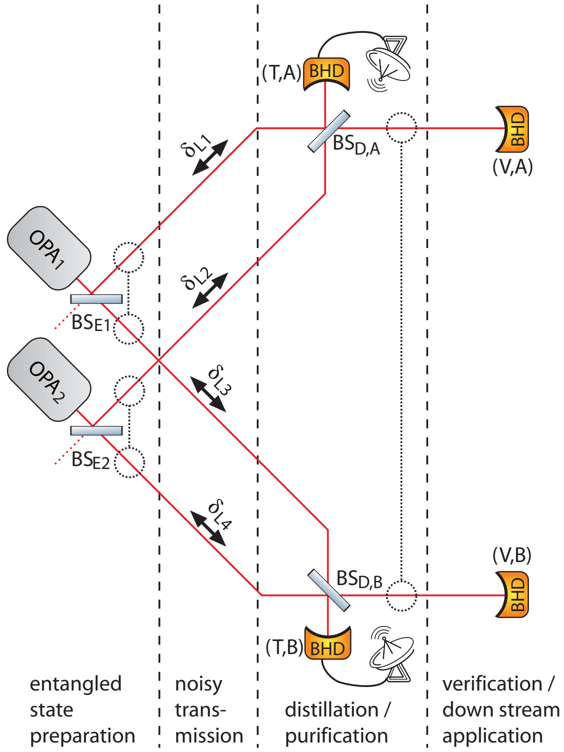

FIG. 1: **Experimental setup.** OPA: optical parametric amplifier, BS$_{\rm D}$: distillation beam splitter, BS$_{\rm E}$: entangling beam splitter, BHD: balanced homodyne detector. $\delta_{\rm Li}$ indicate independently fluctuating optical path lengths. The combination of both BHD$_{\rm T,A/B}$ provided the trigger signal for successful distillation, whereas BHD$_{\rm V,A/B}$ were used for its independent verification. The latter were not necessary for the distillation protocol to work.

Quantum information makes use of the special properties of quantum states in order to improve the quality of communication and information processing tasks. Generally, a quantized field can be described by the number operator or alternatively by two non-commuting position and momentum-like operators. The corresponding measurement results have either discrete or continuous spectra and form the basis of discrete variable (DV) or continuous variable (CV) quantum information, respectively. In the regime of continuous variables, entangled states of light can be deterministically generated in optical parametric amplifiers (OPAs), precisely manipulated with linear optics, and measured with very high efficiency in balanced homodyne detectors. These entangled two-mode squeezed states show Gaussian probability distributions and were utilized for quantum teleportation[12] and entanglement swapping[13,14]. Entangled states of the collective spins of two atomic ensembles analogous to two-mode squeezed states have been generated[15], storage of quantum states of light in an atomic memory has been demonstrated[3] and teleportation from light onto an atomic ensemble has been reported[16]. High-speed quantum cryptography with coherent light beams and homodyne detection has been demonstrated[17]. All these spectacular achievements reveal the great potential of this approach to quantum information processing.

A missing piece in this toolbox has been a feasible protocol for entanglement distillation and purification. Entanglement distillation[4,5] extracts from several shared copies of weakly entangled mixed states a single copy of a highly entangled state using only local quantum operations and classical communication between the two parties sharing the states. This turned out to be a very challenging task for CV states since it was proved that it is impossible to distill Gaussian entangled states by means of the experimentally accessible Gaussian operations[18,19]. However, a whole class of important decoherence processes give rise to non-Gaussian noise and therefore produce non-Gaussian entangled states. It has been shown[9] that in this case the entanglement distillation can be carried out using only interference on beam splitters, balanced homodyne detection and conditioning on the measurement outcomes. This was recently experimentally confirmed by successful demonstrations of distillation and purification protocols for single squeezed modes that suffered from de-Gaussifying noise[20,21,22].

This work experimentally demonstrates the first distillation protocol for CV entangled states. Our protocol uses two copies of phase-diffused[22,23], therefore mixed, entangled states exhibiting a positive, but non-Gaussian Wigner function. Our protocol enhances the entanglement and purity of the decohered states and represents a single step of an iterative Gaussification scheme[7,8] that asymptotically converts any input state into a Gaussian one. Moreover, if combined with a single de-Gaussifying operation such as the recently demonstrated single-photon subtraction from squeezed beams[24,25,26], it would provide a generic continuous variable entanglement purification and distillation scheme[7,8], that is capable, for instance, of suppressing the detrimental effect of losses in quantum state transmission.

A schematic sketch of our protocol is shown in Figure 1. Two optical parametric amplifiers (OPAs) provided two continuous wave light fields that carried squeezed states of light. Both squeezed states were mixed with vacuum states on beam splitters with 50% power reflectivity in order to prepare two copies of so-called v-class[27] entangled states. Entanglement prepared in this way is not the strongest possible but is effectual for this proof of principle experiment. All four resulting beams of the two copies of entangled states were transmitted to two parties Alice (A) and Bob (B) through four channels exhibiting independent phase noise. The noisy channels were realized by quasi-random electro-mechanical actuation of mirror positions in the beam paths[22,23] in order to mimic the phase noise introduced for example in optical fibres. The phase fluctuations applied exhibited a Gaussian distribution, hence the standard deviation of the noise $\sigma_{\rm pn}$ provided a complete characterization of its strength.

Alice and Bob each received two beams which they overlapped on a balanced beam splitter ($\rm BS_{D,A}$ and $\rm BS_{D,B}$ in Fig. 1). The mean phase was controlled such that the initial quadrature phases lined up. The two output ports of each beam splitter were connected to a total of four balanced homodyne detectors (BHDs). Each of the BHDs could be set to observe an arbitrary quadrature, in particular the amplitude quadrature X or the conjugate phase quadrature P. The detectors $\rm BHD_{T,A}$ and $\rm BHD_{T,B}$ were used to generate trigger events discriminating the success or failure of preparation a disitlled state, while $\rm BHD_{V,A}$ and $\rm BHD_{V,B}$ were used to independently verify the performance of the protocol. The latter two were therefore not part of the actual protocol and could be replaced by an arbitrary experiment. The fringe visibilities on all four BHDs as well as on ($\rm BS_{D,A/B}$) were between 97.1% and 98.2%. In our experiment both trigger detectors $\rm BHD_{T,A/B}$ were set to measure the initially squeezed amplitude quadratures $X_{T,A}$ and $X_{T,B}$, respectively. We chose the trigger condition determining the success of the distillation protocol in the form[9]

$$|X_{T,A} + X_{T,B}| < Q, \qquad (1)$$

where Q was a certain threshold whose value could be tuned in order to vary the selectivity of the protocol.

Figure 2a) shows the variance of the nonlocal EPR-like quadrature operator $(X_{V,A} + X_{V,B})$ of the successfully distilled state and its corresponding probability of successful preparation versus threshold value Q for three different strengths of the phase fluctuations $\sigma_{\rm pn}$. The lower the threshold was set, the more selectively our protocol worked. The variance of the orthogonal EPR-like quadrature operator $(P_{V,A} - P_{V,B})$ was also reduced (not shown). In this case the effect was rather small and limited by the vacuum noise value of 1/2, because v-class[27] entangled states were used in our experiment.

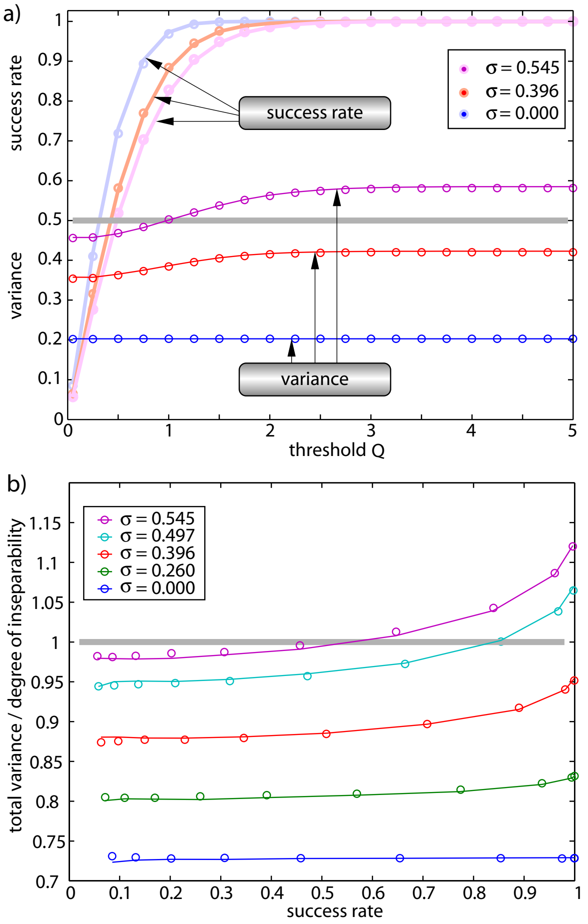

FIG. 2: **Nonlocal quadrature variance and total variance of the distilled states.** (a) Variance of $(X_{V,A} + X_{V,B})$ and corresponding success rates versus threshold value Q for different strengths of the phase fluctuations $\sigma_{\rm pn}$. ○ : measurement, – : simulation. (b) The total variance $\mathcal{I}$ plotted against the success rate can be seen as measure for the performance of our protocol. The grey lines indicate the vacuum noise level or separability boundary, respectively. For a success rate as high as 0.5 the protocol deploys nearly its full potential.

Figure 2b) shows the total variance $\mathcal{I}$ of the distilled state versus its preparation success rate for five different strengths of phase noise. The total variance quantifies the quantum cor-

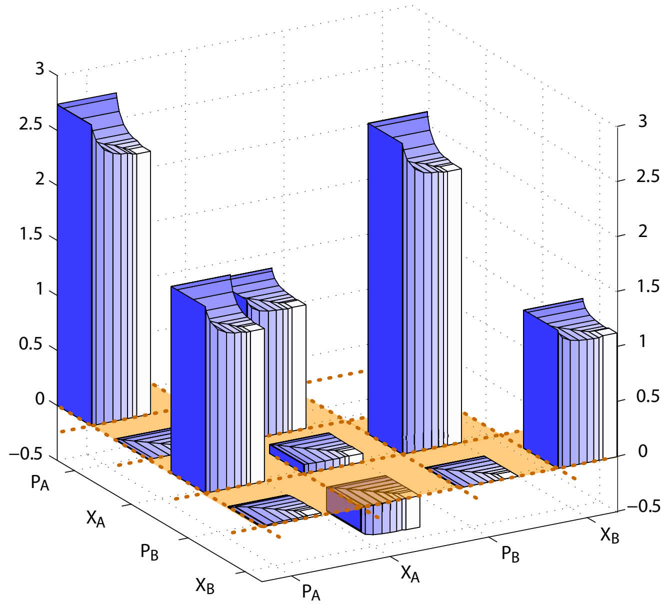

FIG. 3: **Reconstructed covariance matrices of distilled entangled states.** Displayed is the lower left triangle of this matrix containing its ten significant elements. The sub-bars represent the results of our distillation protocol for different threshold values Q in the case of $\sigma_{\rm pn} = 0.497$. The base area of each sub-bar represents the corresponding success rate. The vacuum reference for this plot is given by the unit matrix.

relations between two modes and the degree of inseparability of the joint state (see Methods). If $\mathcal{I} < 1$ then the state is not separable and therefore entangled[28]. We can see that the distillation and purification protocol reduces the total variance $\mathcal{I}$ of the phase-diffused state. For the two strongest levels of phase noise, the total variance shows no nonclassical behaviour without the distillation protocol, but was reduced below the unity boundary after applying the distillation protocol. Lower values of Q result in a stronger distillation effect but also in a reduced success probability. It is promising to find that for a success rate as high as 0.5 the protocol had nearly developed its full potential. Note that the total variance of the entangled state before applying phase noise was evaluated to be $\mathcal{I} = 0.725$.

Since the homodyne detectors can measure arbitrary quadrature phases, we performed a tomographic reconstruction of the covariance matrices of the distilled two-mode entangled states. We followed the reconstruction procedure that is described in detail in our previous work[27] and is based on joint measurements of quadrature operators of modes V,A and V,B for four different settings of the BHDs. From this data we determined the most significant eight of the ten independent parameters of the covariance matrix, namely the variances of four quadratures $X_{\rm V,A}$, $P_{\rm V,A}$, $X_{\rm V,B}$, $P_{\rm V,B}$ and covariances between all pairs of quadratures of Alice and Bob. The intra-modal correlations were neglected, because the deviation from zero was too small to be measured within the given accuracy of the quadrature phase. The resulting covariance matrices are plotted in Fig. 3 for ten different success rates.

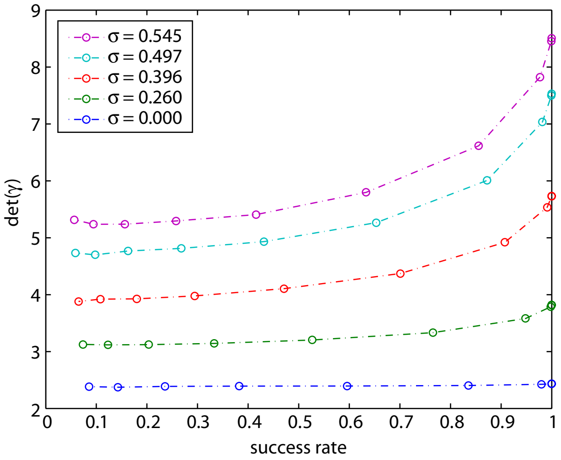

FIG. 4: **Determinant of covariance matrix of distilled state** The determinant D is plotted versus the success rate of the distillation protocols for five different levels of phase noise. The decrease of the determinant D towards the left is an indication of increased purity and Gaussification of the distilled state.

With decreasing success rate the distillation becomes stronger as witnessed by the reduction of the quadrature variances, i.e. diagonal elements of the covariance matrix. Moreover, the anti-correlation between $X_{\rm V,A}$ and $X_{\rm V,B}$ was slightly enhanced. Consequently, the squeezing of the nonlocal quadrature $X_{\rm V,A} + X_{\rm V,B}$ was enhanced by the distillation.

The determinant $D = \det(\gamma)$ is an important characteristic of a covariance matrix γ. The purity $\mathcal{P}$ of a Gaussian state ρ defined as $\mathcal{P} = {\rm Tr}[\rho^2]$ is given by $\mathcal{P} = 1/\sqrt{D}$. In Fig. 4 we plot the dependence of the determinant D on the success rate of the distillation protocol. The distillation reduces D which is a very strong indication of the increased purity of the (still slightly non-Gaussian) distilled state, because a generalized Heisenberg uncertainty relation implies that $D \geq 1$ and $D = 1$ holds only for pure Gaussian states. The anticipation that the distilled states should not only show purification but also Gaussification was indeed confirmed by fitting Gaussian functions to the measurement data before and after the distillation protocol.

The protocol demonstrated here is able to distill entangled states from a decohered ensemble that suffered from phase noise in the transmission channel e.g. due to thermally excited refractive index fluctuations or Brillouin scattering. Because of its quantum nature, our protocol can counteract phase diffusion in cases where any classical channel probing would fail, for example if the phase fluctuations are intensity dependent and arise from quantum fluctuations in the photon number of the transmitted states. Our protocol provides two *open* ports that output the distilled entangled states, and is therefore unconditionally useful for arbitrary down-stream quantum information applications that involve the second order moments of quadrature operators. An example is a teleportation protocol that teleports Gaussian states[12]. The reported entanglement distillation, purification and Gaussification protocol can be iterated[7,8] and combined with already experimentally demonstrated single-photon subtraction[24,25,26], quantum memory[3] and entanglement swapping[13,14] to build a continuous-variable quantum repeater. Our experiment is thus an important enabling step towards truly long-distance quantum communication with continuous variables.

METHODS

Generation and detection of squeezed states Both optical parametric amplifiers were constructed from second order nonlinear crystals (MgO:LiNbO$_3$) inside a degenerate doubly resonant cavity[23] and were pumped with frequency doubled laser beams from a monolithic solid state laser (Nd:YAG) operated at $1064\,$nm. The OPAs emitted continuous wave squeezed beams with about 4.5 dB of squeezing and 8 dB anti-squeezing at Fourier frequencies around 7 MHz.

All four output beams of the distillation protocol were measured by balanced homodyne detectors (BHDs) each consisting of a balanced beam splitter, a coherent local oscillator beam at 1064 nm and two photodiodes. The photocurrents produced by the photodiodes were subtracted and amplified by home-made electronics. The signals from the four BHDs were electronically mixed with a 7 MHz local oscillator. The intermediate signals were then anti-alias filtered at 400 kHz, synchronously sampled with 1 MHz and post-processed to accomplish and verify the distillation protocol.

Entanglement detection and quantification The entanglement of the distilled two-mode squeezed output state can be conveniently certified by the Duan criterion[28]. This criterion involves the sum of variances of two nonlocal, EPR-like quadrature operators, the so-called total variance, $\mathcal{I} = \langle(\Delta X_{+,\mathrm{V}})^2\rangle + \langle(\Delta P_{-,\mathrm{V}})^2\rangle$, where $X_{+,\mathrm{V}} = X_{\mathrm{V,A}} + X_{\mathrm{V,B}}$ and $P_{-,\mathrm{V}} = P_{\mathrm{V,A}} - P_{\mathrm{V,B}}$. The state is entangled if $\mathcal{I} < 1$, where the vacuum state corresponds to a value of $\mathcal{I} = 1$. The total variance $\mathcal{I}$ provides a quantitative measure of the quantum correlations between the quadratures of the distilled states. For symmetric two-mode squeezed Gaussian states $\mathcal{I}$ fully quantifies the entanglement of the state[29]. It has been shown numerically[9] that even for the non-Gaussian phase diffused squeezed states there is a one-to-one correspondence between the reduction of the total variance and the increase of the logarithmic negativity, which is a computable entanglement measure[30].

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Competing Interests The authors declare that they have no competing financial interests.

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[1] Briegel, H.-J., Dür, W., Cirac, J. I. & Zoller, P. Quantum Repeaters: The Role of Imperfect Local Operations in Quantum Communication. *Phys. Rev. Lett.* **81**, 5932 (1998).

[2] Duan, L.-M., Lukin, M. D., Cirac, J. I. & Zoller, P. Long-distance quantum communication with atomic ensembles and linear optics. *Nature (London)* **414**, 413 (2001).

[3] Julsgaard, B., Sherson, J., Cirac, J. I., Fiurášek, J. & Polzik, E. S. Experimental demonstration of quantum memory for light. *Nature (London)* **432**, 482 (2004).

[4] Bennett, C. H. *et al.* Purification of Noisy Entanglement and Faithful Teleportation via Noisy Channels. *Phys. Rev. Lett.* **76**, 722 (1996).

[5] Deutsch, D. *et al.* Quantum Privacy Amplification and the Security of Quantum Cryptography over Noisy Channels. *Phys. Rev. Lett.* **77**, 2818 (1996).

[6] Zukowski, M., Zeilinger, M. A., A. Horne & Ekert, A. K. 'Event-ready-detectors' Bell experiment via entanglement swapping. *Phys. Rev. Lett.* **71**, 4287 (1993).

[7] Browne, D. E., Eisert, J., Scheel, S. & Plenio, M. B. Driving non-Gaussian to Gaussian states with linear optics. *Phys. Rev. A* **67**, 062320 (2003).

[8] Eisert, J., Browne, D. E., Scheel, S. & Plenio, M. B. Distillation of continuous-variable entanglement with optical means. *Ann. Phys.* **311**, 431 (2004).

[9] Fiurášek, J., Marek, P., Filip, R. & Schnabel, R. Experimentally feasible purification of continuous-variable entanglement. *Phys. Rev. A* **75**, 050302(R) (2007).

[10] Pan, J.-W., Gasparoni, S., Ursin, R., Weihs, G. & Zeilinger, A. Experimental entanglement purification of arbitrary unknown states. *Nature* **423**, 417 (2003).

[11] Zhao, Z., Yang, T., Chen, Y.-A., Zhang, A. N. & Pan, J.-W. Experimental Realization of Entanglement Concentration and a Quantum Repeater. *Phys. Rev. Lett.* **90**, 207901 (2003).

[12] Furusawa, A. *et al.* Unconditional Quantum Teleportation. *Science* **282**, 706 (1998).

[13] Takei, N., Yonezawa, H., Aoki, T. & Furusawa, A. High-Fidelity Teleportation beyond the No-Cloning Limit and Entanglement Swapping for Continuous Variables. *Phys. Rev. Lett.* **94**, 220502 (2005).

[14] Jia, X. *et al.* Experimental Demonstration of Unconditional Entanglement Swapping for Continuous Variables. *Phys. Rev. Lett.* **93**, 250503 (2004).

[15] Julsgaard, B., Kozhekin, A. & Polzik, E. S. Experimental long-lived entanglement of two macroscopic objects. *Nature (London)* **413**, 400 (2001).

[16] Sherson, J. F. *et al.* Quantum teleportation between light and matter. *Nature (London)* **443**, 557 (2006).

[17] Grosshans, F. *et al.* Quantum key distribution using gaussian-modulated coherent states. *Nature* **421**, 238 (2003).

[18] Eisert, J., Scheel, S. & Plenio, M. B. Distilling Gaussian States with Gaussian Operations is Impossible. *Phys. Rev. Lett.* **89**, 137903 (2002).

[19] Giedke, G. & Cirac, J. I. Characterization of Gaussian operations and distillation of Gaussian states. *Phys. Rev. A* **66**, 032316

(2002).

[20] Glöckl, O., Andersen, U. L., Filip, R., Bowen, W. P. & Leuchs, G. Squeezed-State Purification with Linear Optics and Feedforward. *Phys. Rev. Lett.* **97**, 053601 (2006).

[21] Heersink, J. *et al.* Distillation of Squeezing from Non-Gaussian Quantum States. *Phys. Rev. Lett.* **96**, 253601 (2006).

[22] Franzen, A., Hage, B., DiGuglielmo, J., Fiurášek, J. & Schnabel, R. Experimental Demonstration of Continuous Variable Purification of Squeezed States. *Phys. Rev. Lett.* **97**, 150505 (2006).

[23] Hage, B. *et al.* On the distillation and purification of phase-diffused squeezed states. *New J. Phys.* **9**, 227 (2007).

[24] Ourjoumtsev, A., Tualle-Brouri, R., Laurat, J. & Grangier, P. Generating Optical Schrödinger Kittens for Quantum Information Processing. *Science* **312**, 83 (2006).

[25] Neergaard-Nielsen, J. S., Nielsen, B. M., Hettich, C., Mølmer, K. & Polzik, E. S. Generation of a Superposition of Odd Photon Number States for Quantum Information Networks. *Phys. Rev. Lett.* **97**, 083604 (2006).

[26] Wakui, K., Takahashi, H., Furusawa, A. & Sasaki, M. Photon subtracted squeezed states generated with periodically poled KTiOPO$_4$. *Opt. Express* **15**, 3568 (2007).

[27] DiGuglielmo, J., Hage, B., Franzen, A., Fiurášek, J. & Schnabel, R. Experimental characterization of Gaussian quantum-communication channels. *Phys. Rev. A* **76**, 012323 (2007).

[28] Duan, L.-M., Giedke, G., Cirac, J. I. & Zoller, P. Inseparability Criterion for Continuous Variable Systems. *Phys. Rev. Lett.* **84**, 2722 (2000).

[29] Giedke, G., Wolf, M. M., Krüger, O., Werner, R. F. & Cirac, J. I. Entanglement of Formation for Symmetric Gaussian States. *Phys. Rev. Lett.* **91**, 107901 (2003).

[30] Vidal, G. & Werner, R. F. Computable measure of entanglement. *Phys. Rev. A* **65**, 032314 (2002).