# Image reconstruction in scalar wave optics

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**Abstract:** Image reconstruction in scalar wave optics is considered in analogy to quantum state reconstruction. Detected intensities correspond to the probabilities. Maximum likelihood estimation is used for measurement of correlation function of an input scalar field.



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Optics was established as a science to acquire information through observation and measurement. This effort is, however, always affected by errors of various natures. There are tight analogies between scalar wave optics and nonrelativistic quantum mechanics. Paraxial Fresnel approximation of Helmholtz wave equation in wave optics is nothing else than Schrödinger equation. Field may be described by coherent scalar amplitude, or more generally, by a second order correlation function (mutual intensity) in the case of partially coherent fields,

$$\Gamma(x,x',z) = \langle \psi^*(x',z)\psi(x,z)\rangle, \qquad \Gamma(x,x',z) = \Gamma^*(x',x,z), \qquad \Gamma(x,x,z) \geq 0.$$

The brackets denote the averaging over complex amplitudes and $z$ being the longitudinal coordinate of the optical setup. For the sake of simplicity a one dimensional transverse space will be assumed (coordinate $x$). Quantum counterparts may be easily established in the Dirac notation. Complete knowledge may be represented using a pure state, whereas any randomness in the ensemble of identically prepared particles corresponds to the incoherent mixture of pure states – a density operator

$$\widehat{\rho} = \sum_k \lambda_k |\varphi_k\rangle\langle\varphi_k|, \qquad \widehat{\rho}^+ = \widehat{\rho}, \qquad \langle\psi|\widehat{\rho}|\psi\rangle \geq 0.$$

The second order correlation function may be interpreted as $x$-representation of density operator $\mathrm{Tr}[\widehat{\rho}|x'\rangle\langle x|] = \Gamma(x,x',z)$. Transformation of complex amplitude in wave optics is given by integral transformation. This corresponds to the coordinate representation of a generic transformation of quantum state,

$$|\psi\rangle = \widehat{T}|\psi_0\rangle, \qquad \langle x|\psi\rangle = \int \mathrm{d}x_0 \langle x|\widehat{T}|x_0\rangle\langle x_0|\psi_0\rangle.$$

The states $|x_0\rangle$ are related to the position in the object plane, whereas $|x\rangle$ to the position in the image plane. The kernel of the integral transformation is the propagator

$$\mathcal{K}(x,x_0) = \langle x|\widehat{T}|x_0\rangle$$

corresponding to the response function (point spread function) in scalar wave optics. Loosely speaking it relates the point source in the object plane ($z = 0$) with its image in the image plane. This mapping is fuzzy in realistic image processing due to the effect of diffraction. The explicit form of response function is fully specified by the geometrical parameters of the optical device. In the case of realistic observations with finite aperture, operator $\widehat{T}$ is not unitary and observations are not orthogonal. Detection represents the last step necessary for establishing the full analogy. Intensity of partially coherent wave in the position $(x,z)$ is given by diagonal elements of mutual intensity $I(x,z) = \Gamma(x,x,z)$. This quantity is measurable and may be sampled using relative frequencies $f_i$. It plays the role of probability in quantum theory $p(x) = I(x)$.

The analogy between scalar wave and quantum theories will be used for the formulation of the following inverse problem. Assuming an unknown signal propagating through optical elements, the output field may be detected. Provided that properties of optical system and properties of detection process are known, the input signal should be inferred from the measurements performed on the output. This scheme has already been considered in analogy with quantum tomography [1]. However, adopting this procedure for realistic noisy data one always runs into nonphysical results, since the inversion is an ill posed problem. To overcome this difficulty the maximum likelihood estimation was proposed for various quantum reconstruction schemes [2, 3, 4]. It has all the features of genuine quantum measurement. This technique will be adopted here for reconstruction of correlation function which yields with the highest likelihood the detected data. Ultimate limitation of image processing beyond the standard resolution will be investigated [5].

The numerical simulation shows the feasibility of the method. The object state created by double slit illuminated by coherent light is measured for twenty discrete transverse displacements of measuring apparatus. This is an ideal thin lens with finite aperture in imaging arrangement and intensity detector consisting from forty discrete pixels in image plane. The correlation function of input state is reconstructed as hermitian matrix $100 \times 100$. Real and imaginary parts of the estimated mutual intensity are shown in the Figs. 1 and 2. The separation of the two slits is two times below the Rayleigh limit producing intensity in image plane as plotted in the Fig. 3. Due to the anisoplanasy of the setup the peaks could be distinguished provided that transverse shift is induced. Reconstructed intensity in the object plane exhibits two distinguishable peaks as shown in the Fig. 4.

Fig. 1

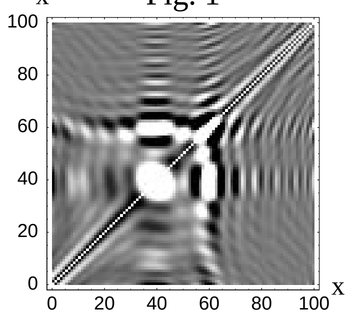



Fig. 2

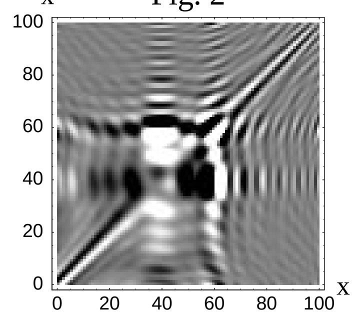



Fig. 3

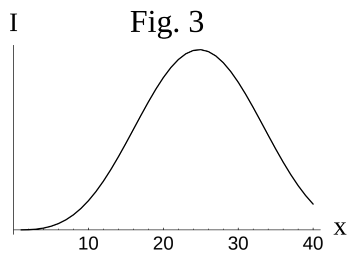
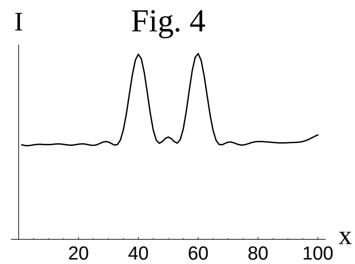



Fig. 4

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