Conditional preparation of arbitrary superpositions of atomic Dicke states

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Atomic state manipulation is essential for many quantum information protocols.

Possible applications

- quantum memory
- quantum repeaters
- quantum computation with atoms
Our goal

Proposal of scheme capable of preparation of arbitrary superpositions of atomic Dicke states.

Target Dicke states

\[ |\psi_{\text{target}}\rangle = \sum_{n=0}^{N} c_n |n\rangle \]

- state of atomic collective spin
- Dicke state \( |n\rangle \) - \( n \) excited atoms
- using light beam to manipulate the atomic state
State of collective atomic spin

- atomic cloud in magnetic field
- collective spin oriented in one main direction
- spin deviations described using $x_A, p_A$ quadratures
- similarly $x_L, p_L$ used for light quadratures
QND interaction

\[ \hat{H}_{\text{QND}} = \hbar \bar{\kappa} \hat{x}_A \hat{x}_L \]
Main scheme

Resulting action

Procedure:
- light in non-Gaussian state
- QND interaction: $\hat{H}_{\text{QND}} = \hbar \kappa \hat{x}_A \hat{x}_L$
- homodyne detection of $\hat{p}_L$

Resulting action on atoms

$$\hat{\Theta}(p_L) \propto (\hat{x}_A + p_L / \kappa) \exp \left[ -\epsilon (\hat{x}_A + p_L / \kappa^2) \right]$$

- in Dicke basis: $|0_A\rangle \rightarrow \cos(\alpha)|0_A\rangle + \sin(\alpha)|1_A\rangle$
- $\alpha$ depends on homodyne detection outcome
Combination with displacement

- basic QND interaction is enveloped by displacement operations on atoms
- displacement can be implemented by magnetic field
- allows further control over the resulting atomic state
to prepare atomic state containing $|n\rangle$ Dicke state, one needs to repeat the basic QND interaction $N$ times.

allows preparation of superpositions of Dicke states with only real roots in the wave function’s polynomial part.
Fidelity - probability trade off

- homodyne detection outcome is random
- atomic action $\hat{\Theta}(p_L)$ depends on HD outcome
- average fidelity $F(\ket{\psi_{\text{prepared state}}}, \ket{\psi_{\text{desired target state}}}) < 1$

- more HD outcomes accepted → higher scheme success probability
  → lower average fidelity
Numerical simulations

Numerical simulation

\( \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \) (red): average fidelity about 90% even for 100% success probability

\( |2\rangle \) (blue): lower average fidelity due to two repetitions of the basic QND interaction scheme
Which HD outcomes shall we accept?

- the best strategy = best fidelity - probability trade off
- plot fidelity as a function of HD outcomes (contours)
- accept every HD outcome within chosen fidelity contour
Alternative schemes

Basic QND scheme generalised

- addressing rotated atomic quadrature $\hat{x}_{\theta A}$
- additional degree of freedom: $\theta$

Generalised interaction

$$\hat{H}(\theta)_{\text{QND}} = \hbar \kappa \hat{x}_{\theta A} \hat{x}_L$$

- not limited only to "real roots"
- capable of preparation of any superposition of Dicke states
Alternative strategy

\[ \hat{U}_{\text{QND}} = \exp^{i\kappa \hat{x}_A \hat{x}_L} \] similar to Fourier transform

- idea: prepare the light in a specific state
- the QND interaction imprints the desired state on atoms
- easy to calculate which state of light one needs
- only one QND interaction with atoms is necessary
- light state may be difficult to prepare
Conclusion

- basic scheme for Dicke states preparation proposed
- QND interaction accompanied by displacement operation
- limited to "real roots" superpositions
- several numerical simulations and strategy optimization
- rotated atomic quadrature scheme overcomes the "real roots" limitation
- Fourier based approach, only one interaction needed
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Thank you for your attention.