Photonic simulation of quantum systems: combating noise and weak interaction

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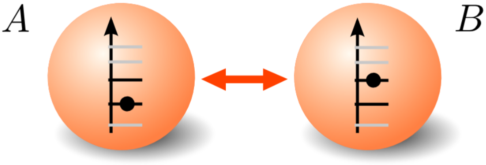
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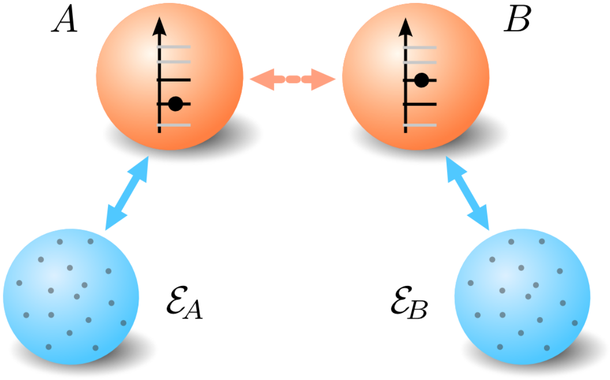
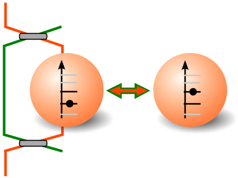
Interaction of quantum systems under real conditions

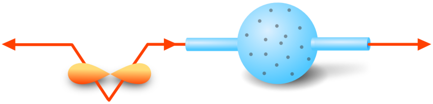


Outline of the talk

- Quantum C-Z gate for weakly interacting qubits
  – boosting interaction by interference
  – photonic simulator



- Entanglement transfer through noisy environment
  – multi-particle incoherent environment
  – environment probing
  – photonic simulators



Outline of the talk

1. Quantum C-Z gate for weakly interacting qubits

2. Entanglement transfer through noisy environment

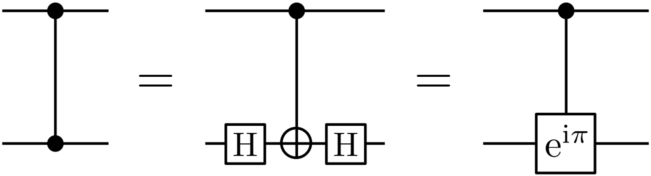
Quantum C-Z and C-PHASE gates

CZ$= I - 2\,|11\rangle\langle 11|$

CZ $|jk\rangle = (-1)^{jk}\,|jk\rangle$

CPHASE$= \exp\left(\mathrm{i}\phi\,|11\rangle\langle 11|\right)$

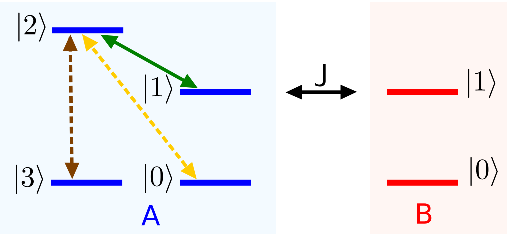
$\phi = \pi$



spin-spin coupling

Quantum C-Z gate of weakly spin-spin coupled qubits

$U_\phi = \exp(\mathrm{i}\phi\,|11\rangle\langle 11|)$
$\phi < \pi$



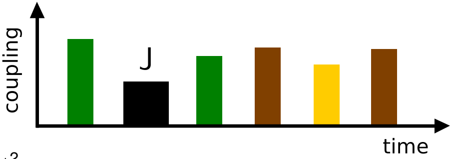
Auxiliary BS coupling $|1\rangle \rightleftarrows |2\rangle$

$|1\rangle \to t|1\rangle + r|2\rangle$
$|2\rangle \to \tilde{t}^*|2\rangle - \tilde{r}^*|1\rangle$

Projection back to $\{|0\rangle, |1\rangle\}$

$(t\tilde{t} - r\tilde{r}^*)|0\rangle\langle 0| + |1\rangle\langle 1|$

CZ: $r\tilde{r}^*/t\tilde{t} = (1 + e^{i\phi})/2$



Probability of success: $P_s = |t\tilde{t} - r\tilde{r}^*|^2$
maximized for $|t|^2 = |\tilde{t}|^2 = 1/[1 + |\cos(\phi/2)|]$

[A. Feizpour et al., PRL 107, 133603 (2011); C. Simon and E.S. Polzik, PRA 83, 040101(R) (2011)]

Beam splitter coupling of two bosonic modes

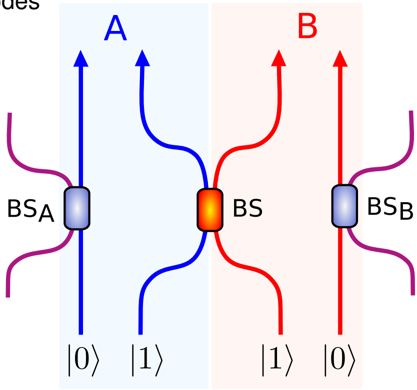
$$U_{BS}(t) = \exp\left[\kappa\, t\,(ab^{\dagger} - a^{\dagger}b)\right]$$

$$U_{BS}(R) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-R} & 0 & 0 \\ 0 & 0 & \sqrt{1-R} & 0 \\ 0 & 0 & 0 & 1-2R \end{pmatrix}$$

Strong interaction

$R = \sin^2(\kappa t) > 1/2$

$U_{BS}(2/3) \sim \mathrm{CZ}$



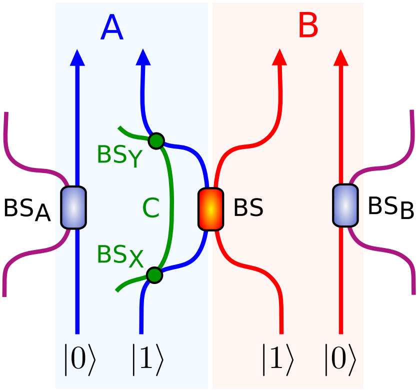
Quantum C-Z gate of weakly BS-coupled qubits

Weakly coupled systems
$R = \sin^2(\kappa t) < 1/2$

BS partially bypassed by C
$BS_{X,Y}$: amp. transmittances t_X, t_Y

C-Z: $\frac{r_X r_Y}{t_X t_Y} = \frac{3R-2}{2\sqrt{1-R}}$

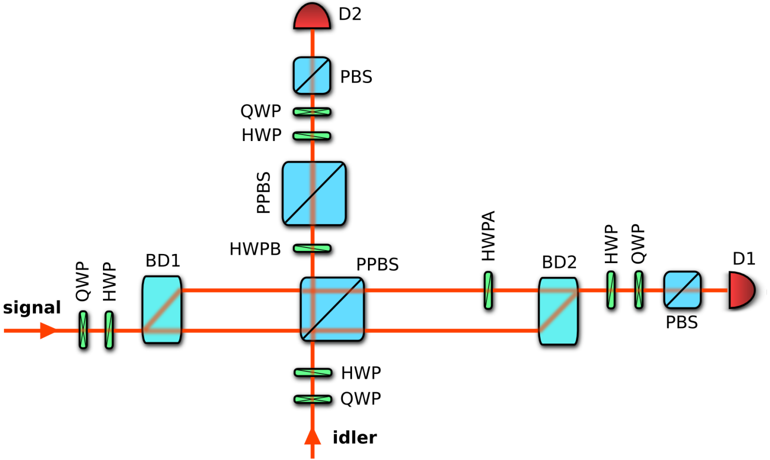
Probability of success: $R^2 \frac{T_X T_Y}{4}$
maximized for $T_X = T_Y$



$$U = \begin{pmatrix} t_A t_B & 0 & 0 & 0 \\ 0 & t_A t & 0 & 0 \\ 0 & 0 & t_B(t t_X t_Y + r_X r_Y) & 0 \\ 0 & 0 & 0 & (2t^2-1)t_X t_Y + t r_X r_Y \end{pmatrix} = R\frac{t_X t_Y}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Photonic simulator of C-Z gate with weak coupling and bypass

BS: PPBS $R_V = 1/3,\ R_H = 0$



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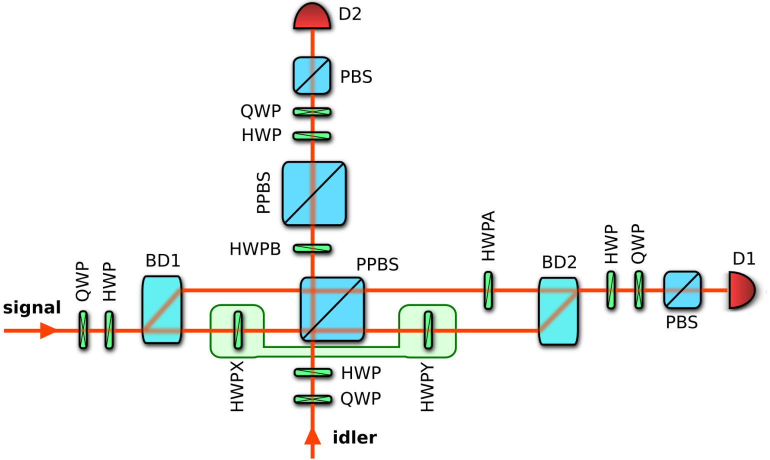
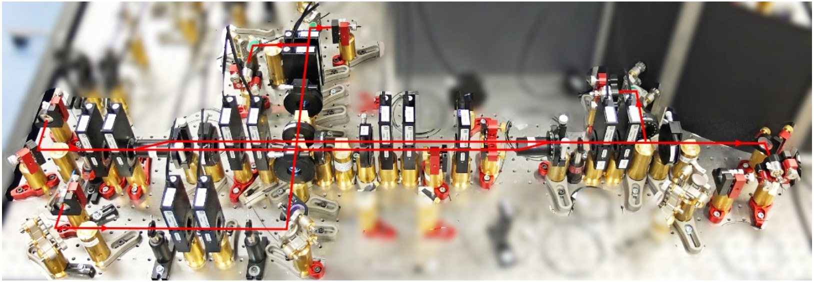


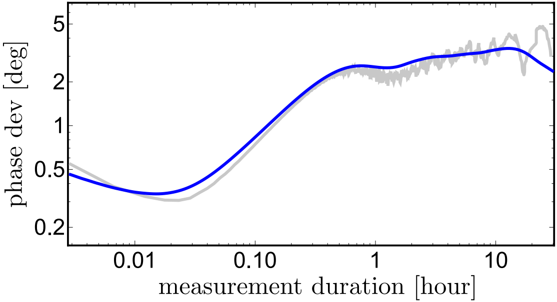
BS$_{X,Y}$ bypass: HWPX and HWPY, $T_X = \cos^2(2\phi_X)$

Photonic simulator of C-Z gate with weak coupling and bypass

Experimental setup based on Toffoli gate experiment

[M. Mičuda et al., Phys. Rev. Lett. 111, 160407 (2013)]



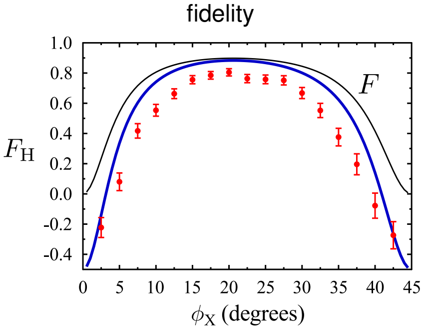


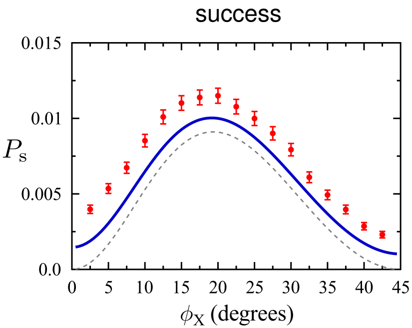
Simulation results

Gate fidelity: $F_\chi = \mathrm{Tr}[\chi\chi_{\mathrm{CZ}}]/(\mathrm{Tr}[\chi]\mathrm{Tr}[\chi_{\mathrm{CZ}}])$

Hofmann bound on quantum process fidelity: $F_\chi \geq F_1 + F_2 - 1$

[H.F. Hofmann, PRL 94, 160504 (2005); M. Mičuda et al., PRA 89, 042304 (2014)]





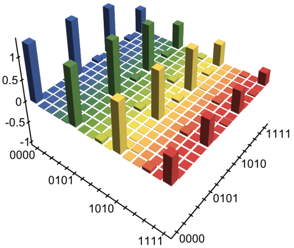
Theoretical model: $R_H = 1.9\%$, $\mathcal{V}_{\mathrm{HOM}} = 94\%$

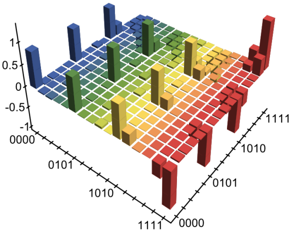
Maximum predicted gate fidelity: 88.9% at $\phi_X = 20^\circ$

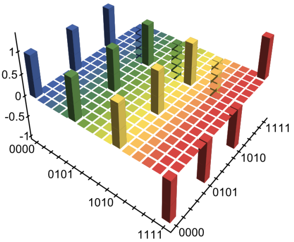
Simulation results: full tomography of quantum process

Number of preparation/measurement configurations: $6^4 = 1296$
10 s acquisition per configuration $\rightarrow$ total time of 6 hours

without bypass	optimal bypass	ideal C-Z gate







$F_\chi = 84.6\%$

[M. Mičuda, R. Stárek, I. Straka, M. Miková, M. Dušek, M. Ježek, R. Filip, and J. Fiurášek, submitted 2015]

Outline of the talk

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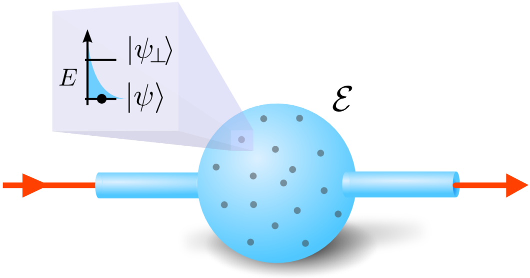
2. Entanglement transfer through noisy environment

Noisy environment

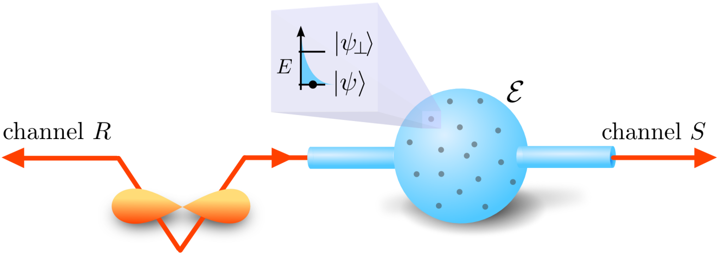
Multi-qubit environment $\mathcal{E}$:

$$\rho = (1 - p_T)|\psi\rangle\langle\psi| + p_T|\psi_\perp\rangle\langle\psi_\perp| \qquad p_T = \frac{\exp\left(-\frac{\Delta E}{k_B T}\right)}{1+\exp\left(-\frac{\Delta E}{k_B T}\right)}$$



Incoherent environment: qubits do not interfere or interact with another qubits

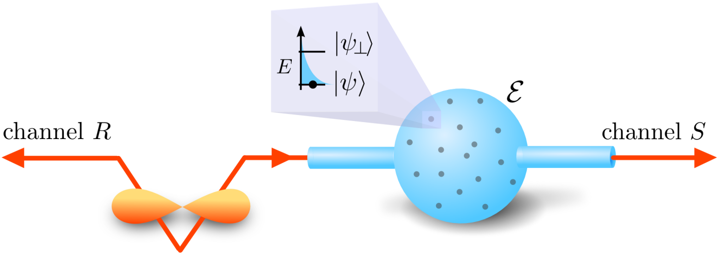
Entanglement transfer through incoherent environment



Input state $|\Psi^-\rangle = \frac{1}{\sqrt{2}}\left(|\psi\rangle|\psi_\perp\rangle - |\psi_\perp\rangle|\psi\rangle\right)$

Output state $\rho_{RS} = P_S|\Psi^-\rangle_{RS}\langle\Psi^-| + (1 - P_S)\frac{1_R}{2} \otimes \rho_S$

Entanglement transfer through incoherent environment



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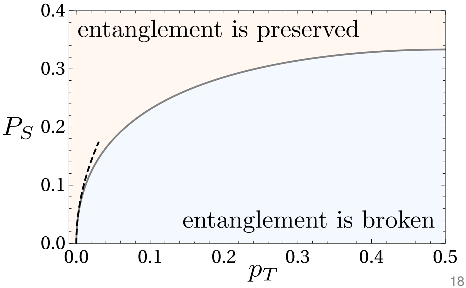
Output state $\rho_{RS} = P_S|\Psi^-\rangle_{RS}\langle\Psi^-| + (1-P_S)\frac{1_R}{2}\otimes\rho_S$

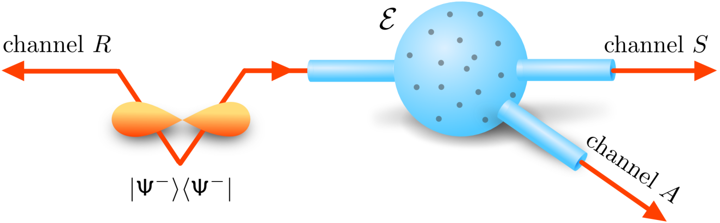
The state remains entangled if

$$P_S > \frac{\sqrt{p_T(1-p_T)}}{1+\sqrt{p_T(1-p_T)}}$$

$$\frac{p_T}{P_S^2} < 1 \quad \text{for} \quad p_T \ll 1$$



Three possible processes: success, flip, and error; $P_S + P_F + P_E = 1$

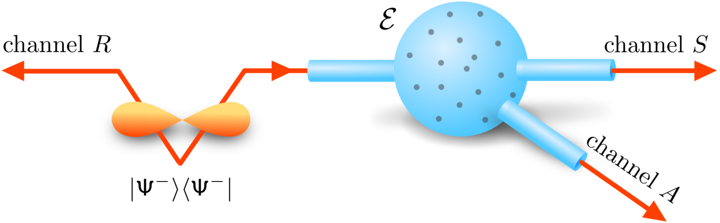


Projecting channel A to environment's ground state gives

$$\rho_{RS} \propto (1-p_T)P_S|\Psi^-\rangle_{RS}\langle\Psi^-| + \frac{P_F}{2}|\psi_\perp\rangle_R\langle\psi_\perp| \otimes \rho_S + (1-p_T)P_E\frac{1_R}{2} \otimes \rho_S$$

Using additional environment probing

Three possible processes: success, flip, and error; $P_S + P_F + P_E = 1$



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The state remains entangled if

$$P_S > \frac{1}{2}\left(\sqrt{p_T P_E(4 - 3p_T P_E)} - p_T P_E\right)$$

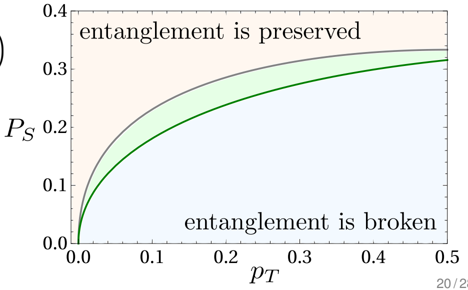
$$\frac{p_T P_E}{P_S^2} < 1 \quad \text{for} \quad p_T \ll 1$$

$P_E = 0$ for single particle environment

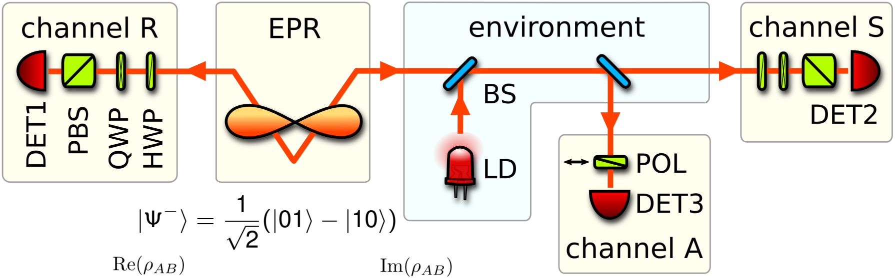
[E. Nagali et al., PRA 79, 060304(R) (2009)]

[M. Gavenda et al., PRA 81, 022313 (2010)]

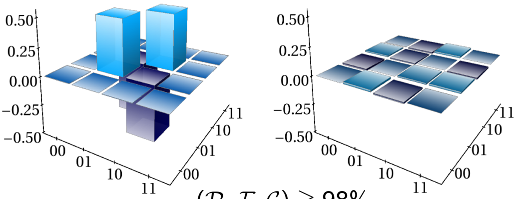
[M. Gavenda et al., PRA 83, 042320 (2011)]



Photonic simulator



$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$



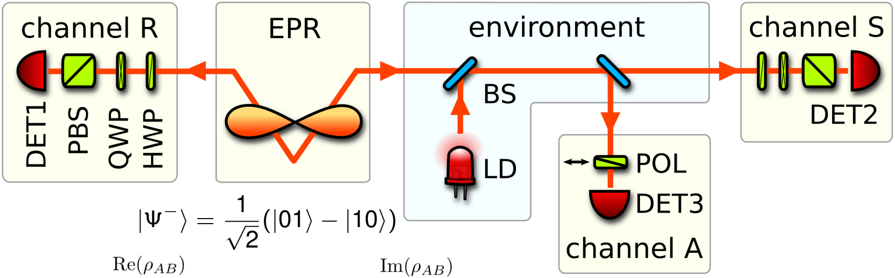
$(\mathcal{P}, \mathcal{F}, \mathcal{C}) \gtrsim 98\%$

[M. Miková et al., PRA 90, 022317 (2014)]

$|0\rangle\langle 0|$ conditioning (or Tr)

- 3-fold coincidences
- Full tomography
- Coincidence window
- Noise depolarization

Photonic simulator



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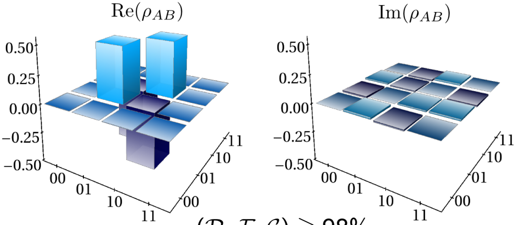

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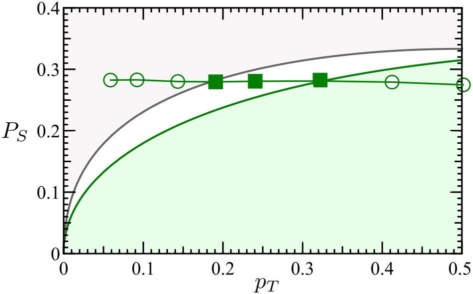
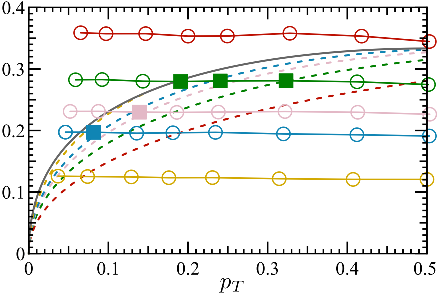
Model of the simulator:

$$\rho_{RS} \propto (1-p_T)|\Psi^-\rangle_{RS}\langle\Psi^-| + \frac{1}{2}|1\rangle\langle 1| \otimes \rho_S + (1-p_T)\widetilde{P}_E \frac{1_R}{2} \otimes \rho_S, \quad \widetilde{P}_E \propto \frac{\tau R_S R_N}{R_{\psi^-}}$$

$P_E = 0.42$

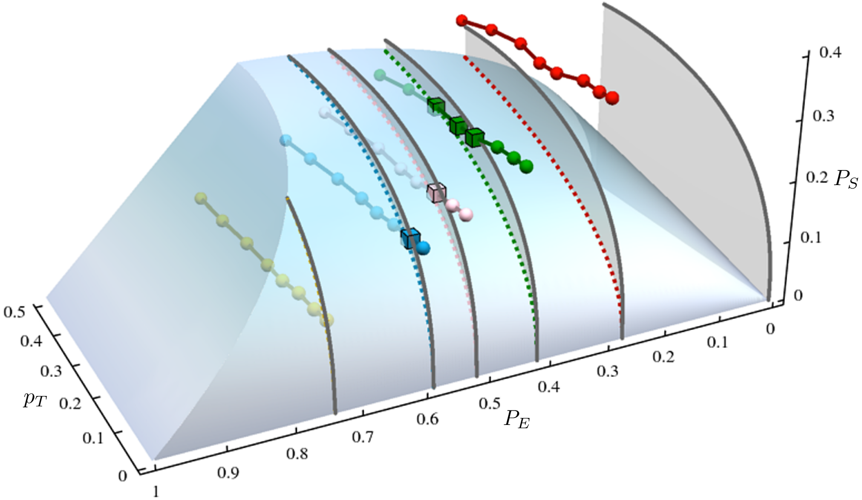


$P_E = 0.27, 0.42, 0.52, 0.58, 0.74$



Simulation rusults and generalization of the simulator

Model: $\rho_{RS} \propto (1-p_T)|\Psi^-\rangle_{RS}\langle\Psi^-| + \frac{1}{2}|1\rangle_R\langle 1| \otimes \rho_S + (1-p_T)\widetilde{P}_E \frac{1_R}{2} \otimes \rho_S$



[I. Straka, M. Miková et al., in preparation 2015]

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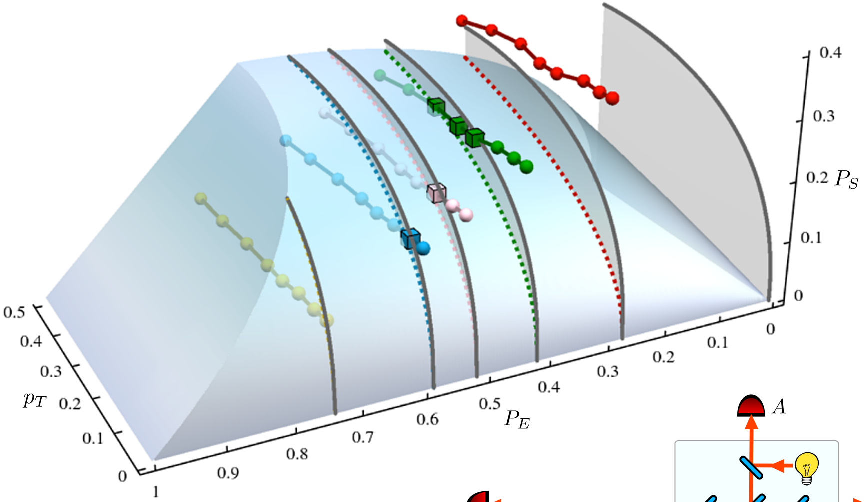

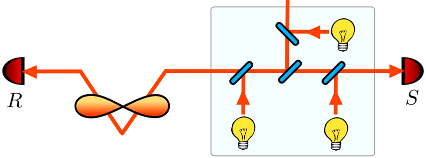

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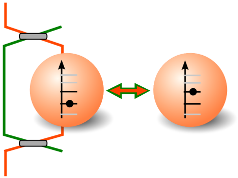

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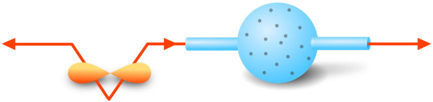








Thank you for your attention!








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