Journal of Modern Optics
Vol. 53, Nos. 5–6, 20 March–15 April 2006, 689–697

Tomographic analysis of vortex information content

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(*Received 1 February 2005; in final form 14 July 2005*)

Non-diffracting vortex beams are perspective information carriers for free space communications enabling an effective encoding, transfer and decoding of information. Adopting coaxial vortex superpositions, additional degrees of freedom of light are available to increase the density of the carried information. In this paper we present a novel decoding scheme motivated by a quantum state estimation technique capable of extracting full information encoded into an array of Bessel–Gauss beams.

1. Introduction

Light is a convenient information carrier. In conventional information channels based on a temporal modulation, a sequence of N optical pulses is used to transfer N bits of information. Adopting optical vortex fields, additional degrees of freedom of light are available for increasing the density of the carried information. In the recently proposed protocols [1, 2], the transferred information is encoded into the light spatial structure so that M bits of information can be transferred by each optical impulse. The optical vortex beam can be comprehended as a specific light field whose equiphase surface has the form of a helical spiral with a pitch of $m\lambda$, where m is an integer called the topological charge of the vortex and λ denotes the vacuum wavelength. At the vortex centre, the phase exhibits singularity and the intensity vanishes. The electromagnetic energy rotates around the vortex point and creates a non-zero orbital angular momentum [3]. Generation of the vortex arrays composed of the spatially separated vortices with different topological charges is also possible. It was verified that such a vortex field is structurally stable in a region whose length can be controlled by geometrical parameters of the optical elements used [4]. In an advanced experimental set-up, generation of vortex arrays was realized by means of a spatial light modulator (SLM) [5]. By this experiment, a mixed vortex field representing a coherent superposition of the coaxial vortices can also be created. As has been recently shown [1, 2], the mixed vortex field can be used as a perspective information carrier

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Journal of Modern Optics
ISSN 0950–0340 print/ISSN 1362–3044 online © 2006 Taylor & Francis
http://www.tandf.co.uk/journals
DOI: 10.1080/09500340500259920

because it enables an effective encoding, transfer and decoding of information. The individual vortices comprising the mixed vortex field are superposed with weight coefficients whose values 1 or 0 create an information chain. Due to its non-diffracting character, the mixed vortex field then propagates as a structurally stable beam to a receiver. The topological charges of the vortices are used as 'markers' enabling their spatial separation and the decoding of information at the receiver. This is done applying a complex-valued phase mask and subsequent intensity detection by the CCD camera. The vortices, whose weight coefficients are set to 1, are detected in the transversal plane as bright spots appearing in predetermined positions, while the positions corresponding to vortices with zero weights remain dark. In this paper, an advanced method enabling the tomographic analysis of vortex information content is proposed and experimentally verified. Instead of using transforming masks (i.e. hardware) for decoding purposes, as was done in [1, 2], the proposed approach relies on a proper reconstruction analysis (i.e. software) of detected images at the receiving station.

The paper is organized as follows. In section 2 the theory of mixed vortex beams is briefly reviewed. The decoding scheme utilizing SLM is presented in section 3. The tomography of vortex beams is explained in detail in section 4. Finally, computer simulation of the proposed information protocol and main experimental results are presented in section 5.

2. Mixed vortex beams

A creation of pseudo-non-diffracting (P-N) mixed vortex beams follows from the integral representation of a general monochromatic P-N beam whose complex amplitude ψ is given by [6]

$$\psi(\mathbf{r}) = \int_0^{2\pi} A(\gamma) g(\mathbf{r}, \theta, \gamma) \mathrm{d}\gamma. \tag{1}$$

In that treatment, the P-N field propagating along the z axis is expressed as a superposition of inclined Gaussian beams $g(\mathbf{r}, \theta, \gamma)$ whose axes create a conical surface with the vertex angle 2θ. The z direction is the cone axis. The axes of the separate Gaussian beams used in the superposition are specified by an azimuthal angle γ. The complex amplitudes of the superposed Gaussian beams can be modified by an arbitrary periodic function $A(\gamma)$. By a convenient choice of A, integral (1) results in an array of spatially separated vortices or in a coaxial superposition of vortex beams with different topological charges [1]. In the simplest way of information encoding, the coaxial vortex beams are used. The superposition of M coaxial vortex beams with the topological charges m is obtained if A is of the form

$$A(\gamma) = \sum_{m=1}^{M} \mathrm{i}^m a_m \exp(\mathrm{i} m \gamma), \tag{2}$$

where a_m denotes the weight coefficients. The integration (1) performed with the azimuthal modulation (2) results in [1]

$$\psi(\mathbf{r}, t) = \psi_0(z)G(\mathbf{r}, t)\sum_{m=1}^{M} a_m \psi_m(r, z)\exp(\mathrm{i}m\varphi), \tag{3}$$

where

$$\psi_m(r) = J_m\left[\alpha\left(1 + \frac{z}{q}\right)r\right], \tag{4}$$

$$\psi_0(z) = 2\pi \exp\left[\mathrm{i}\frac{\alpha^2 z}{2k}\left(1 - \frac{z}{q}\right)\right]. \tag{5}$$

Equation (3) describes a coaxial superposition of the Bessel beams carried by a background Gaussian beam G modified by a propagation function u_0. The parameter α defines a spot size of the Bessel beam, r and φ denote the circular cylindrical coordinates and q is the complex parameter of a background Gaussian beam. It can be expressed by the confocal parameter q_0 as $q = z + \mathrm{i}q_0$. The mixed vortex field (3) propagates as the so-called P-N field. Though its intensity profile depends on the propagation coordinate z, its changes are negligible in the propagation region whose length L can be approximated by $L \approx kw_0/\alpha$, where w_0 is the waist radius of the background Gaussian beam and k denotes the wave number. In that region, an experimentally realizable P-N beam represents a good approximation of an ideal non-diffracting beam. In the limit case $w_0 \to \infty$, the coaxial superposition of vortex beams (3) becomes an ideal non-diffracting field whose intensity profile remains unchanged under free propagation,

$$\psi(\mathbf{r}) = \psi_0(z)\sum_{m=1}^{M} a_m J_m(\alpha r)\exp(\mathrm{i}m\varphi), \tag{6}$$

where

$$\psi_0(z) = 2\pi \exp\left[-\mathrm{i}\left(1 - \frac{\alpha^2}{2k^2}\right)z\right]. \tag{7}$$

As is obvious, the number of vortices used in the coaxial superposition and their topological charges and weight coefficients are defined by the modulating function A. In experiments based on an application of the SLM, it can be simply controlled. In this way, the coaxial vortex superposition can be used as an effective information carrier for the free-space communications.

3. Transfer of information by vortex beams

In comparison with common light beams, the mixed vortex field (3) provides additional degrees of freedom applicable to a new way of information encoding. To realize it, an actual base of vortices is defined. It represents a coaxial superposition of M vortices with topological charges m and unitary weights $a_m = 1$,

$\psi = \psi_0 \sum_{m=1}^{M} u_m(r)\exp(\mathrm{i}m\varphi)$. Applying the SLM, an input laser beam is converted into a field given as a weighted superposition of non-diffracting vortex beams,

$$\psi_{a_1, a_2, \ldots, a_M} = \psi_0 \sum_{m=1}^{M} a_m u_m(r)\exp(\mathrm{i}m\varphi). \tag{8}$$

Its weight coefficients with values '1' or '0' create an information code $a_1, a_2, \ldots, a_M$ representing M bits of information. The created field is non-diffracting (in a real case pseudo-non-diffracting) so that it propagates without significant changes toward a receiver. In a decoding, the vortex field is transmitted through a mask whose transparency T is proportional to the complex conjugate base used for the information encoding,

$$T = \sum_{m=1}^{M} u_m^*(r)\exp(-\mathrm{i}m\varphi + \Phi_m). \tag{9}$$

After the Fourier transform, the values '1' of the weight coefficients are detected as bright intensity spots at the positions predetermined by the phase terms Φ_m. For the weight coefficients with the values '0', the related positions remain dark. In this way, the transferred information is decoded.

A scheme of the proposed information channel is shown in figure 1. In the illustrated example, the vortices with the topological charges 1, 3, 5 and 7 ($m = 2m' - 1,\ m' = 1, 2, 3, 4$) enabling an encoding of four information bits are used. An actual information code 1110 means that the mixed vortex field is composed of vortices with the topological charges 1, 3 and 5, the remaining vortex has the zero weight coefficient. The required information code is written into the computer generated hologram sent to the SLM (CRL OPTO 1024×768 pixels). In this case, only the amplitude of the field needs to be modulated. A collimated Gaussian beam impinging on the SLM splits into different diffraction orders and it is directly converted into the predetermined mixed vortex field after a spatial filtering performed by the 4-f Fourier system. The mixed vortex field then propagates

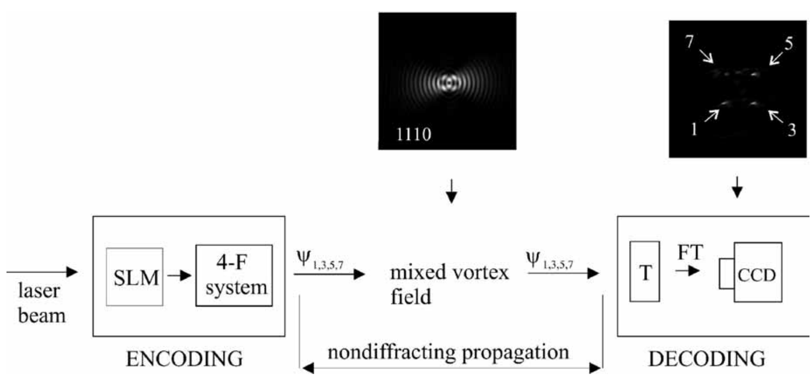

Figure 1. Experimental set-up of the free-space optical information channel based on mixed vortex beams and subsequent phase unwrapping; for description see the text.

through a free space to the receiver without significant structural changes. Furthermore, it revives to an initial form behind amplitude and phase obstacles. In the decoding process, the mixed vortex field is transmitted through a mask whose transparency is defined by the complex conjugate base used for the information encoding. After an optically performed Fourier transform of the transmitted field, the vortices are spatially separated and detected by the CCD camera (640×480 pixels). The separate vortices are identified at the positions defined by the mask used. The vortices with the topological charges 1, 3 and 5 represent bright spots while the position of the vortex with the topological charge 7 remains dark. The actual information code is decoded as 1110.

4. Tomography of the vortex information content

Decoding of information from mixed vortex arrays by phase unwrapping has already been presented in [1, 2]. Here we will focus on the decoding of information from a single intensity scan. Apart from the above-mentioned applications in classical communications, vortex beams have also attracted the attention of the quantum optical community since they provide a relatively easy way of realizing higher dimensional quantum states—qudits [7, 8]. Motivated by this interpretation we will restate the decoding of classical information from vortex arrays as a quantum problem, considering the vortex beams appearing in the superposition equation (8) as a convenient eigenbasis in a truncated Hilbert space. Since the coefficients a_k are arbitrary complex numbers, their decoding represents a quantum state reconstruction problem.

At first it may seem impossible to obtain full information about the coefficients from a single transversal intensity scan. Indeed, such a scan would be just a position measurement, where detections at different pixels would realize commuting (compatible) projections,

$$I(x_j) \approx |x_j\rangle\langle x_j|, \quad \langle x_j|x_k\rangle = 0, \quad \forall j \neq k. \tag{10}$$

Obviously, to determine the full density matrix one has to make mutually incompatible observations as well. So in general, an additional operation is necessary to generate tomographically complete data. Such a transformation is provided, for example, by free propagation, which tends to mix position and momentum observables. The tomography of the transversal coherence matrix from several transversal intensity scans has been discussed in [9]. Let us mention in passing that for optical systems that are not strictly isoplanatic, such a reconstruction can reveal details below the Rayleigh limit and thus yields superresolution.

Returning back to the mixed arrays of Bessel–Gaussian beams one can say that here the situation is much simpler. Note that Bessel beams are not complete and hence span only a small subspace of the total Hilbert space of all possible optical fields. Also, due to their non-diffracting character, intensity scans in different axial positions are equivalent. As will be shown in the following, this makes it possible to determine coefficients a_k from a single intensity scan.

In this paper, we will only discuss the case of positive weights $a_k \geq 0, \forall k$, which is already quite general. The optical field after the SLM can be approximated by the superposition

$$\psi(r, \varphi) = \sum_m a_m \exp(\mathrm{i}m\varphi) J_m(r). \tag{11}$$

The detected intensity will be proportional to the expectation value of its modulus squared which reads as

$$I = \sum_{m,n} \langle a_m a_n^* \rangle J_m(r) J_n(r) \exp[\mathrm{i}(m-n)\varphi]. \tag{12}$$

Now the moduli of coefficients a_k can be obtained for instance by integrating the detected intensity over phase,

$$S(r) = \int_0^{2\pi} I(r, \varphi) \mathrm{d}\varphi = \sum_m \rho_{mm} J_m^2(r), \tag{13}$$

where we denoted ρ_{mm} the diagonal elements of the sought-after coherence matrix. As one can see, the data obtained by integration over phase is a linear combination of the unknown coefficients with known positive weights. This problem is an example of a class of linear and positive problems. Linearity as well as the positivity of all involved quantities is obvious. Although a direct inversion of the linear transformation equation (13) is not always possible due to the dimensions involved and/or physical positivity constraints on ρ_{mm}, one can always solve this problem in a statistical sense using e.g. the well-known nonlinear expectation-maximization algorithm [10, 11],

$$\rho_{mm}^{i+1} = \rho_{mm}^{i} \frac{\sum_j S(r_j)}{\left[\sum_j s(r_j)\right]\left[\sum_m J_m^2(r_j)\right]} \sum_{j=1}^{n_r} \frac{s(r_j) J_m^2(r_j)}{S(r_j)}, \tag{14}$$

where the superscript i labels iterations, $s(r)$ are the experimentally measured values of phase averaged intensities $S(r)$ and all continuous quantities have been replaced by discrete ones.

5. Simulations and experiment

The suggested detection scheme has been tested in simulations and also experimentally. A simulation of noiseless transmission of four vortices ($m = 1, 2, 3, 4$) is shown in figure 2. This figure shows dependence of transmission errors on the size of data used for the reconstruction. As usual in quantum considerations, the fidelity has been adopted here as the figure of merit of the protocol. It quantifies the difference between the reconstructed weights $\mathbf{b}$ and the true weights $\mathbf{a}$ used for the transmission,

$$F = \frac{\mathbf{a} \cdot \mathbf{b}}{[(\mathbf{a} \cdot \mathbf{a})(\mathbf{b} \cdot \mathbf{b})]^{1/2}}. \tag{15}$$

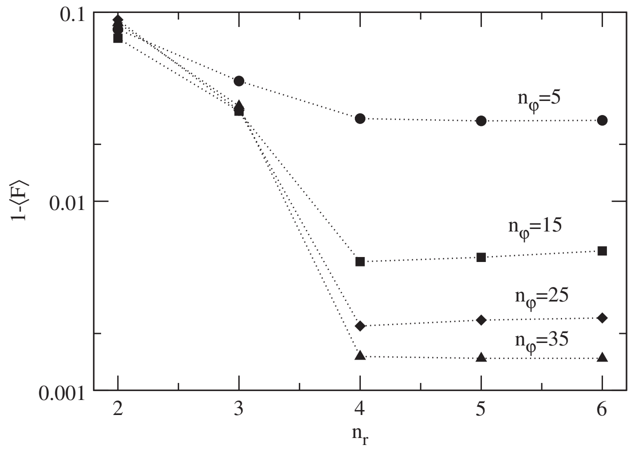

Figure 2. Simulated transmission errors on a logarithmic scale versus the numbers of radii n_r and phases n_φ used for the reconstruction.

Mean fidelity $\langle F \rangle$ then has been calculated by averaging F over a set of several hundred randomly generated input vectors $\mathbf{a}$. In general, mean fidelity depends on the number of phase values n_φ used for evaluating the integrals in equation (13) and also on the number of radii n_r used for inversion via the EM algorithm equation (14). As can bee seen in figure 2, increasing n_r over a certain threshold has little effect on the accuracy of reconstruction. This can be expected as four different phase integrals should be enough to reconstruct four unknown parameters $a_1 \cdots a_4$. On the other hand a large number of phases used for approximating the phase integrals leads to a higher fidelity of transmission. In any case, even small sets of data $n_r = 4$ and $n_\varphi = 15$ yield fidelities over 99%.

The above-presented decoding of the vortex information content has been tested experimentally on vortex arrays consisting of up to four different on-axis vortices prepared using a computer-generated amplitude mask (CRL OPTO 1024×768 pixels). The experimental set-up was similar to that shown in figure 1 with two notable differences: (i) the decoding part was replaced by a tomographic analysis of recorded images and (ii) positive real weights were used instead of just zeroes and ones. Unfortunately, the simple approach based on inverting the phase integrals equation (14), that had performed so well in simulations, was found to be rather sensitive to proper centring as well as distortions of the transferred beams. Therefore, as the first step, we used a more robust, though computationally more demanding inversion of the whole detected image. Here, a particularly efficient inversion method would be the maximum-likelihood estimation, see e.g. [12]. Gaussian approximation of the likelihood function that is valid for intense signals then leads to further simplification, since the maximization of the highly nonlinear likelihood function can be replaced by the standard least-squares fit of the theoretical intensity distribution to the detected transversal intensity scan. This approximation was certainly valid in our experiment, in which the generated vortex fields were

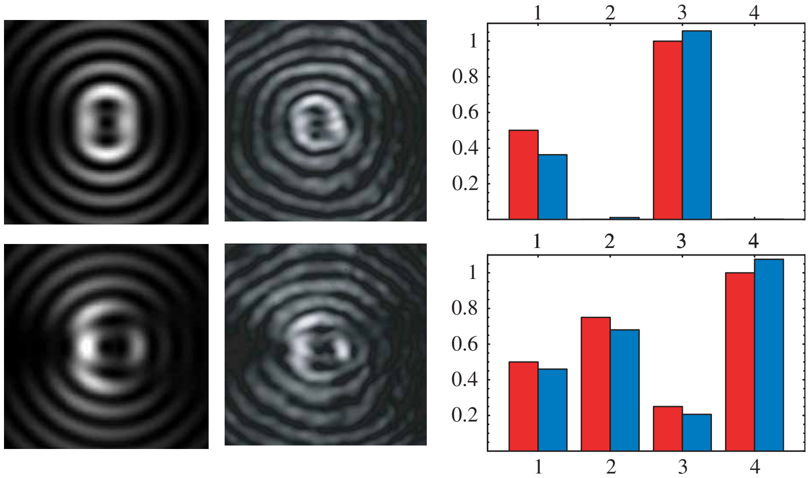

Figure 3. Experimental decoding of the vortex information content. Left column: two different theoretical intensity distributions used for driving the SLM, only amplitude was modulated; middle column: measured CCD scans; right column: reconstructed weights (right bars) as compared to the true weights (left bars) used for the two transmissions. (The colour version of this figure is included in the online version of the journal.)

transmitted over distances of the order of one metre. Apart from being more robust, fitting of the whole transversal intensity distribution has the advantage of being more general: it can be used to decode positive as well as negative, and even complex weight coefficients, increasing further the information capacity of the proposed communication channel.

The experimental results are summarized in figure 3. The first row shows the reconstruction of a vortex field with the corresponding true weights $\mathbf{a} = \{0.5, 0, 1, 0\}$. Only two vortices are populated in this superposition. The second row shows the reconstruction of a more complicated vortex field with four non-zero weights $\mathbf{a} = \{0.5, 0.75, 0.25, 1\}$. As is seen, the presence of even number vortices makes the transversal intensity distribution less symmetric. The reconstructed weight coefficients agree well with those used for driving the spatial light modulator in the preparation stage of the experiment, the corresponding fidelities being over 98% in both cases. In real applications where weak signals are likely to play an important role, the proposed decoding method may need to be refined by replacing the standard least squares by more advanced inverse methods such as the maximum-likelihood estimation. This point is worth further studies.

6. Conclusions

In this paper we have proposed the novel decoding scheme for vortex-based optical communication motivated by the theory of quantum state reconstruction. This analysis is capable of extracting full information encoded into weights of a superposition of Bessel–Gaussian beams. Simple communication based on transmitting up to four vortices simultaneously was verified experimentally.

Acknowledgments

This work was supported by Project MSM6198959213 of the Czech Ministry of Education and Project FT-TA2/059 of the Czech Ministry of Industry and Trade.

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