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Experimental demonstration of a teleportation-based programmable quantum gate

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We experimentally demonstrate a programmable quantum gate that applies a sign flip operation to data qubit in an arbitrary basis fully specified by the quantum state of a two-qubit program register. Our linear-optical implementation is inspired by teleportation-based schemes for quantum computing and relies on multiphoton interference and coincidence detection. We confirm the programmability of the gate and its high-fidelity performance by carrying out full quantum process tomography of the resulting operation on data qubit for several different programs.

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Quantum computers can handle some computational tasks—such as factoring large numbers [1] and searching unsorted data [2]—more efficiently than classical computers can do. Several feasible paradigms of quantum computation have been proposed [3–6] and selected elementary quantum logic gates and processors have been demonstrated [7]. Depending on what computational task is desired, a particular dedicated processor has to be used. By contrast, classical computers offer much more flexibility for a fixed hardware, where the operation to be performed on bits of a data register is determined by a suitable program encoded in a program register.

In a seminal paper, Nielsen and Chuang [8] extended the concept of programmable processors to quantum computation. They considered a universal quantum gate that can be programmed to perform an arbitrary unitary operation on the data qubits. It turns out that, assuming a finite-size quantum program register, such universal gate can be implemented without errors only in a probabilistic fashion [8,9]. Despite their probabilistic nature, programmable gates are remarkable since a complete information on the implemented quantum operation, which requires infinitely many classical bits to specify, can be encoded into a finite number of quantum bits. Recently, a proof-of-principle experimental realization of a probabilistic programmable quantum gate was demonstrated [10], where a single program qubit determines a phase shift applied between fixed basis states of a data qubit [11]. As a natural further step we can consider a programmable gate where the computational basis itself is also specified by the quantum state of the program register. This is equivalent to programming a full single-qubit unitary operation, which requires at least two-qubit program [8].

Here, we report on a linear-optics experimental realization of a high-fidelity programmable quantum gate with a two-qubit program register. The gate can perform a sign flip operation (SIGN) on the data qubit in an arbitrary computational basis $\{|\phi\rangle,|\phi_\perp\rangle\}$ [12],

$$\Sigma_\phi|\phi\rangle=|\phi\rangle, \quad \Sigma_\phi|\phi_\perp\rangle=-|\phi_\perp\rangle, \tag{1}$$

where $\langle\phi|\phi_\perp\rangle=0$. The computational basis in which the gate performs the sign flip is fully specified by the quantum state of the program register. In order to encode a particular basis $\{|\phi\rangle,|\phi_\perp\rangle\}$, the program qubits should be prepared in a symmetric maximally entangled state

$$|\Phi\rangle_\mathrm{p}=\frac{1}{\sqrt{2}}(|\phi\phi_\perp\rangle+|\phi_\perp\phi\rangle). \tag{2}$$

We utilize the scheme of universal quantum computing of Gottesman and Chuang [3] based on generalized quantum teleportation [13] that was recently used to demonstrate teleportation-based controlled-NOT gate [14]. We apply Bell-state measurement (BSM) on the data qubit $|\psi\rangle_\mathrm{d}=\alpha|\phi\rangle+\beta|\phi_\perp\rangle$ and one qubit from the program register, see Fig. 1. Particularly, if the qubits 1 and 2 are projected onto the maximally entangled singlet state

$$|\Psi^-\rangle=\frac{1}{\sqrt{2}}(|01\rangle-|10\rangle)\equiv\frac{1}{\sqrt{2}}(|\phi\phi_\perp\rangle-|\phi_\perp\phi\rangle), \tag{3}$$

then the remaining qubit 3 is prepared in the sign-flipped state $|\psi_\mathrm{out}\rangle_\mathrm{d}=\alpha|\phi\rangle-\beta|\phi_\perp\rangle$. The success probability of the gate is thus 1/4. The success probability of the scheme for programmable SIGN gate proposed in Ref. [12] is 1/3. However, this latter approach requires more complex state of program register and four controlled-NOT gates between data and program qubits. Our implementation is significantly less demanding at a cost of only slightly reduced success rate.

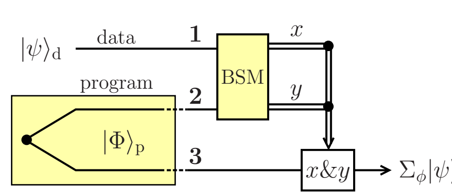

FIG. 1. (Color online) Quantum circuit for teleportation-based programmable quantum logic gate. Time proceeds from left to right. The single wires carry quantum bits (qubits) and the double wires carry classical bits. Qubits 2 and 3 form a program register prepared in a particular entangled state $|\Phi\rangle_\mathrm{p}$. BSM is applied to data qubit 1 in an unknown quantum state $|\psi\rangle_\mathrm{d}$ and qubit 2 of the program register. For singlet Bell state detected in channels 1 and 2 the measurement yields $x=y=1$ which heralds the successful gate operation leaving the remaining qubit 3 in the target state $\Sigma_\phi|\psi\rangle$. The unitary operation Σ_ϕ is determined by the state of the program register.

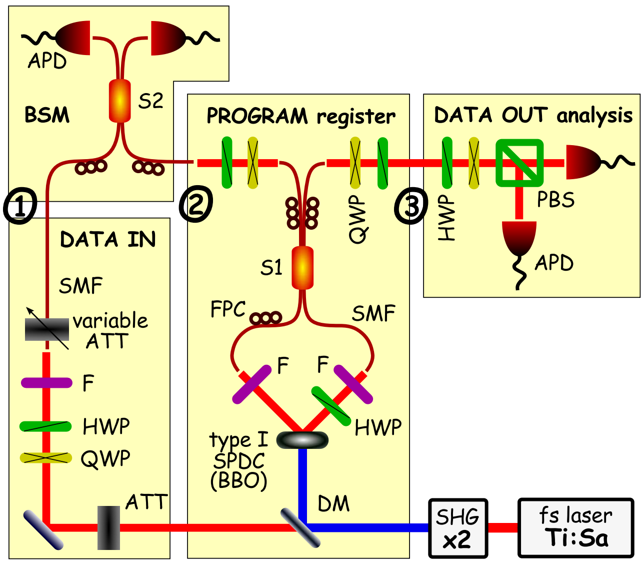

FIG. 2. (Color online) A scheme of the experimental setup. The initial subpicosecond pulses generated by titan-sapphire laser (Ti:Sa) at the fundamental wavelength of 800 nm are frequency doubled to 400 nm by the process of second harmonic generation (SHG) and separated from the fundamental ones by dichroic mirror (DM). The frequency doubled pulses pump a 2-mm-thick nonlinear crystal of β-barium borate (BBO) cut for non-collinear type-I SPDC. The downconverted correlated photons 2 and 3 at 800 nm are filtered by narrow-band 1.75 nm FWHM interference filters (F), coupled to single-mode optical fibers (SMF), and mixed by 50/50 fiber beam splitter (S1) which assures the perfect mode matching of the photon modes. Polarization states of the photons are set to be orthogonal before the splitter. The occurrence of one photon in each output port of the splitter yields the entangled polarization singlet state of the program register. It is further set by half-wave plate (HWP) and quarter-wave plate (QWP) to any chosen entangled state. The data qubit 1 is encoded into a polarization state of a single photon from the attenuated (ATT) fundamental laser pulse, frequency filtered, and coupled to single-mode fiber. The data photon interferes with photon 2 from the program register at the second 50/50 fiber beam splitter (S2) and the coincidence events are detected at the output by single-photon avalanche photodiodes (APDs) and multichannel coincidence logic. A polarization state of the remaining photon 3 carrying output data qubit is simultaneously analyzed using wave plates and polarizing beam splitter (PBS). Fixed unitary polarization transformations induced by optical fibers are compensated by fiber polarization controllers (FPCs). The photon arrival times are precisely synchronized by optical delays not shown in the scheme.

The experimental scheme relies on double Hong-Ou-Mandel interference [15] of three photons generated by separated sources of different photo-statistics [16]. The input data qubit 1 is encoded into a polarization state of a single photon from an attenuated coherent laser pulse, whereas program-register qubits 2 and 3 are carried by polarization states of signal and idler photons prepared by pulsed spontaneous parametric downconversion (SPDC), see Fig. 2. The downconverted pair of photons with perpendicular linear polarizations set by wave plates is coupled into single-mode fibers and interferes at a balanced fiber beam splitter S1 producing the maximally entangled polarization singlet state $|\Psi^-\rangle_{23}$ that is transformed to any desired program-register state [Eq. (2)] easily by local unitary operations. We independently verified the correctness of the program-state preparation by means of full quantum state tomography [17]. Photons 1 and 2 then interfere on the second balanced fiber splitter S2, and they are detected in coincidence basis. The joint detection event represents partial BSM that projects the photons 1 and 2 onto the singlet state $|\Psi^-\rangle$.

The desired SIGN operation of the gate in particular basis is analyzed by input-output probing for several chosen program-register states. The data photon is prepared in one of the four input states $|H\rangle$ (linear horizontally polarized), $|V\rangle$ (linear vertically polarized), $|D\rangle=\frac{1}{\sqrt{2}}(|H\rangle+|V\rangle)$ (linear diagonally polarized), and $|R\rangle=\frac{1}{\sqrt{2}}(|H\rangle+\mathrm{i}|V\rangle)$ (right-circularly polarized). The state of the photon 3 at the output of the gate is measured in three mutually unbiased polarization bases $\{|H\rangle,|V\rangle\}$, $\{|D\rangle,|A\rangle\}$, and $\{|R\rangle,|L\rangle\}$, where $|A\rangle=\frac{1}{\sqrt{2}}(|H\rangle-|V\rangle)$ and $|L\rangle=\frac{1}{\sqrt{2}}(|H\rangle-\mathrm{i}|V\rangle)$. This yields altogether 24 characteristic threefold coincidence rates. The raw data were corrected for the different detection efficiencies of the employed detectors and for accidental counts, which have been both measured prior to the main experiment.

From the experimental data we reconstruct the quantum operation that fully characterizes the transformation of the data qubit for a fixed state of the program register. For this purpose we exploit the Jamiolkowski-Choi isomorphism [18] which tells us that every single-qubit quantum operation $\mathcal{E}$ can be represented by a positive semidefinite operator χ on Hilbert space of two qubits. This operator can be given a clear and intuitive physical meaning; χ is a density matrix of a two-qubit state obtained from the initial maximally entangled state $|\Phi^+\rangle=|HH\rangle+|VV\rangle$ by applying the operation $\mathcal{E}$ to one of the qubits. We have $\chi=\mathcal{I}\otimes\mathcal{E}(|\Phi^+\rangle\langle\Phi^+|)$, where $\mathcal{I}$ denotes the identity operation. Thus χ is a square positive semidefinite Hermitian matrix with four columns and rows. The condition $\chi\geq 0$ is equivalent to the fact that the operation is physical, i.e., that the map $\mathcal{E}$ is completely positive. Given a density matrix of the input state ρ_{in}, the output density matrix can be calculated according to the formula $\rho_{\mathrm{out}}=\mathrm{Tr}_{\mathrm{in}}[(\rho_{\mathrm{in}}^T\otimes\mathbb{1}_{\mathrm{out}})\chi]$, where T denotes transposition in a fixed basis, e.g., $\{|H\rangle,|V\rangle\}$. For each input state ρ_j the probability of a particular measurement outcome associated with projector Π_k on the output state is given by $p_{jk}=\mathrm{Tr}[(\rho_j^T\otimes\Pi_k)\chi]$. From the measured coincidence rates that approximate the theoretical probabilities p_{jk} we determine the quantum process matrix χ by an iterative maximum-likelihood estimation algorithm that is described in detail elsewhere [19]. This statistical reconstruction method yields a quantum process matrix χ that is most likely to produce the observed experimental data and, simultaneously, assures complete positivity of the reconstructed operation.

The input-output measurement and the full quantum process tomography was carried out for five different entangled states of the program register [Eq. (2)] specified by the following single-qubit basis states $|\phi_j\rangle$, $j=1,\ldots,5$,

$$|\phi_1\rangle=|H\rangle,$$

$$|\phi_2\rangle=|R\rangle=\frac{1}{\sqrt{2}}(|H\rangle+\mathrm{i}|V\rangle),$$

$$|\phi_3\rangle = |D\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle),$$

$$|\phi_4\rangle = \frac{1}{\sqrt{4+2\sqrt{2}}}[(1+\sqrt{2})|H\rangle + |V\rangle],$$

$$|\phi_5\rangle = \frac{1}{\sqrt{2}}|H\rangle + \frac{1+\mathrm{i}}{2}|V\rangle. \quad (4)$$

We quantify the performance of the implemented quantum gates by the process fidelity defined as follows:

$$F(\chi,\chi_\phi) = \frac{\mathrm{Tr}[\chi\chi_\phi]}{\mathrm{Tr}[\chi]\mathrm{Tr}[\chi_\phi]}. \quad (5)$$

Here χ_ϕ is a process matrix representing the target SIGN gate Σ_ϕ for a particular basis $\{|\phi\rangle,|\phi_\perp\rangle\}$,

$$\chi_\phi = \mathbb{1} \otimes \Sigma_\phi |\Phi^+\rangle\langle\Phi^+| \mathbb{1} \otimes \Sigma_\phi^\dagger. \quad (6)$$

Note that χ_ϕ is effectively a density matrix of a pure maximally entangled two-qubit state.

A detailed analysis of the reconstructed process matrices χ_j, $j=1,\ldots,5$, reveals that they exhibit an identical small unitary deviation $\delta\Sigma$ from the programmed operation Σ_ϕ due to insufficiently compensated polarization transformation induced by optical fibers employed in BSM. This deviation can be corrected by applying the inverse unitary operation $\delta\Sigma^\dagger$ on the input photon 1 using polarization controller or numerically applying the fixed correcting unitary operation to all the reconstructed process matrices. The optimal correcting operation $\delta\Sigma^\dagger$ can be determined by maximizing the average process fidelity $\bar{F}=\frac{1}{5}\Sigma_j F(\widetilde{\chi_j,\chi_{\phi_j}})$, where

$$\tilde{\chi}_j = (\delta\Sigma \otimes \mathbb{1})\chi_j(\delta\Sigma^\dagger \otimes \mathbb{1}). \quad (7)$$

We parametrize the SU(2) matrix $\delta\Sigma$ by the three Euler angles and numerically determine their optimal values which maximize the average process fidelity $\bar{F}$. We emphasize that since the unitary operation $\delta\Sigma$ is fixed, it does not influence the programmability of the gate in any way.

The final reconstructed quantum processes corrected for the fixed unitary offset $\delta\Sigma$ are shown in Fig. 3. The process fidelity determined from the reconstructed process matrices is shown in Table I. The average process fidelity is 0.90 ± 0.03 which demonstrates very good functionality and performance of the programmable SIGN gate. Without the correction for $\delta\Sigma$, the average fidelity of 0.85 ± 0.03 is reached. Another important characterization of the implemented gates is provided by the purity of the effective two-qubit state χ, $\mathcal{P}=\mathrm{Tr}[\chi^2]/(\mathrm{Tr}[\chi])^2$. The purity quantifies how close is the transformation to a purity-preserving operation $\mathcal{E}(\rho)=E\rho E^\dagger$. The resulting values of $\mathcal{P}$ given in Table I exceed 0.79, and this further confirms that the operations applied by the programmable gate are close to purity-preserving unitary transformations. Finally, we also calculate the entanglement of formation E_f of the process matrix χ. This quantity characterizes the ability of the programmable gate to preserve entanglement. In particular, when the gate is applied to one photon from a maximally entangled pair then E_f will be the resulting entanglement of that photon pair. For all the reconstructed process matrices it holds that $E_f > 0.71$, cf. Table I, so the programmable gate is capable to preserve a large fraction of entanglement when applied to one part of an entangled state.

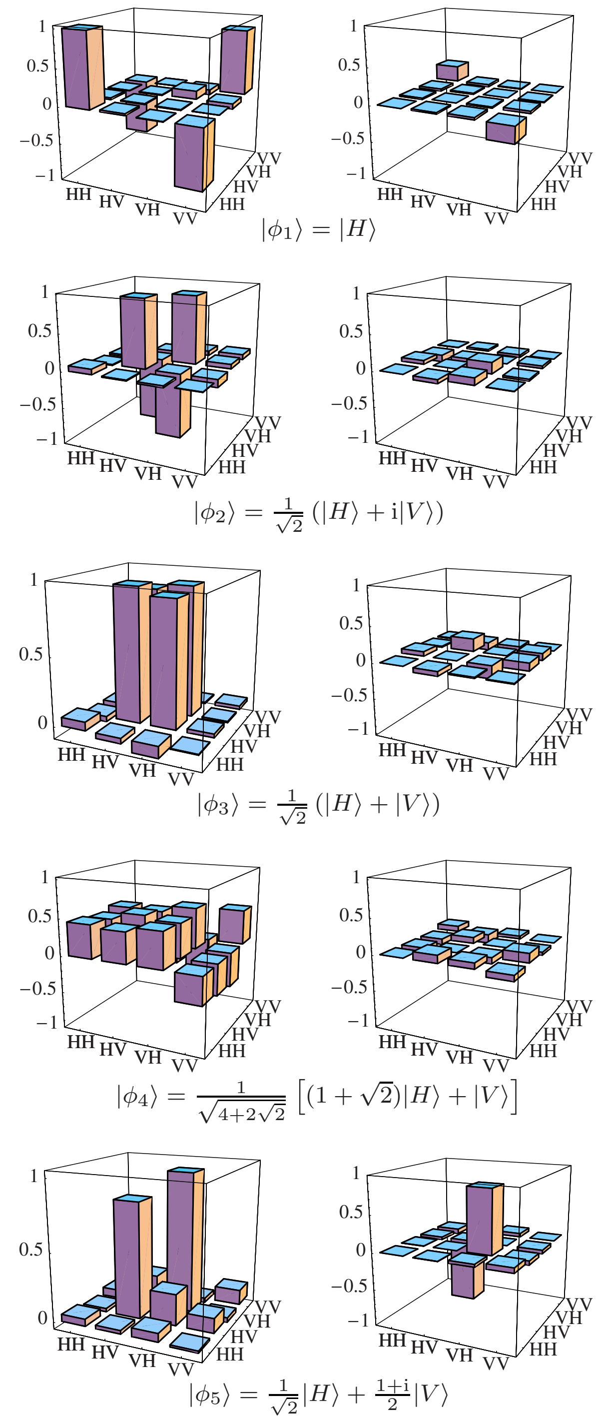

FIG. 3. (Color online) The complete characterization of the programmable SIGN gate operation by means of full quantum process tomography. The real (left column) and imaginary (right column) parts of the reconstructed process matrix χ are shown for five different program-register states [Eq. (2)] encoding the sign flip operation in bases $\{|\phi_j\rangle,|\phi_{j\perp}\rangle\}$, with $j=1,\ldots,5$.

TABLE I. Parameters of the implemented programmable SIGN gate determined from the reconstructed process matrices shown in Fig. 3 and corresponding to five different programs specified by basis states $|\phi_j\rangle$. Shown are the quantum process fidelity F that attains the average value 0.90 ± 0.03, the process purity $\mathcal{P}$ and the entanglement of formation E_f whose average values read 0.85 ± 0.04 and 0.80 ± 0.06, respectively.

Basis	$\|\phi_1\rangle$	$\|\phi_2\rangle$	$\|\phi_3\rangle$	$\|\phi_4\rangle$	$\|\phi_5\rangle$
F	0.866	0.903	0.931	0.905	0.889
$\mathcal{P}$	0.788	0.844	0.902	0.852	0.837
E_f	0.712	0.798	0.889	0.816	0.780

In summary, we have experimentally implemented a programmable single-qubit quantum gate which can perform a rotation of a Bloch sphere about π radians about an arbitrary axis specified by the quantum state of a program register. We have performed a complete tomographic characterization of the gate for several different programs and thus confirmed its programmability and very good performance. To the best of our knowledge, this is the first demonstration of a programmable quantum gate with a two-qubit register that is necessary for the considered class of encoded operations. Our scheme can be straightforwardly generalized to fully universal programmable gate that can apply arbitrary unitary operation on the data qubit and it paves the way toward realization of even more complex programmable quantum gates with multiphoton programs. On more fundamental basis, our experiment confirms the striking capability of few-qubit quantum states to contain complete information on a quantum operation whose classical characterization would require infinitely many classical bits. These unique features of quantum states can be explored in teleportation-based quantum computation [3], and our work can be seen as an important enabling step in this direction.

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