

# Speed and Kinetic Energy of Relativistic Electrons\*†

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Using a Van de Graaff electrostatic generator and a linear accelerator, the speeds of electrons with kinetic energies in the range 0.5–15 MeV are determined by measuring the time required for the electrons to traverse a given distance. The measurements show the existence of a limiting speed in accord with the results of special relativity. The kinetic energy, determined by calorimetry, verifies that an electric field exerts a force on a moving electron in its direction of motion that is independent of its speed.

## INTRODUCTION

ONE of the familiar features of the special theory of relativity is the existence of a universal limiting speed equal to the speed of light  $c$ . This is inherent in the relativistic transformation equations and is illustrated by the law of composition of velocities. The dependence of the speed of a particle on its kinetic energy exhibits this limiting speed and also displays the Newtonian relation as an approximation appropriate at low speeds.

In the past few decades, the successful operation of multi-MeV accelerators and the planning and interpretation of many high-energy physics experiments have affirmed and illustrated the results of special relativity. However, because these efforts generally involved many other concepts, the relations of relativity are not demonstrable from the data in a simple quantitative manner. The purpose of this paper is to describe an experiment that was performed to help fill the need for such simple and direct data for the teaching of relativity. The experiment made use of the electron linear accelerator facility at MIT to provide electrons with kinetic energies between 0.5 and 15 MeV. The speed of the electrons was determined from a measurement of the time

required for the electrons to traverse a given distance. The kinetic energy of the electrons determined from the measurement of accelerating electric fields and potentials was also directly measured by calorimetry.

## THE SPEED MEASUREMENTS

The apparatus is shown schematically in Fig. 1. A pulsed beam of electrons from a Van de Graaff electrostatic generator is injected into an electron linear accelerator (Linac). A fast thyatron pulser at the high-voltage end of the Van de Graaff generator causes the electron gun to emit electrons in a burst lasting about  $3 \times 10^{-9}$  sec, and it does this periodically 120 times per sec. This pulser is triggered by a phototube that in turn is driven by a light flash generated outside the Van de Graaff tank. The electrons emerge from the Van de Graaff in bursts of roughly trapezoidal shape in time (see insert, Fig. 1).

A short insulated metal tube (about 10 cm long) is located between the Van de Graaff and the Linac. This tube collects some of the electrons in each burst passing through it and a voltage pulse is produced which signals the start of the burst down the flight path (the evacuated metal pipe that forms the Linac). At the far end of the Linac the electrons are stopped in an aluminum disk. The charging of this disk produces a second signal. Measuring the time interval between these two signals gives the time required by the electron burst to traverse the 8.4 m length of the drift tube.

This time of flight is determined from the separation of these two signals as displayed on an oscilloscope. The two signals, one from the beginning and one from the end of the flight path,

\* The experiment was performed at the Electron Linear Accelerator facility of the Laboratory for Nuclear Science, MIT.

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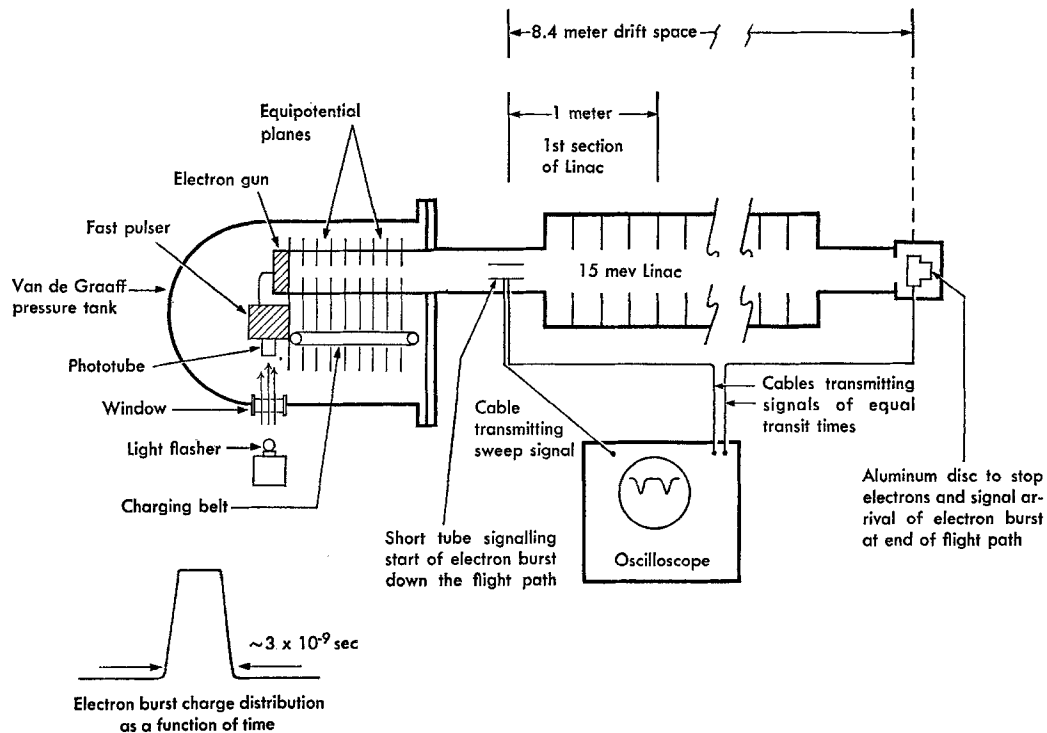


FIG. 1. Schematic diagram of the experiment set up for measuring the time of flight of the electron burst from the Van de Graaff.

are transmitted along two separate cables with equal transit times. The signals are added electrically, preserving their relative positions in time, and are presented to the single input of a Tektronix 581 oscilloscope. Photographs of the scope face for all the different energies used in this experiment are shown in Fig. 2. The data taken from the photographs are shown in Table I.

TABLE I. The measured displacement of the signals and the resulting time for the beam to traverse the 8.4 m flight path are given for each of the energies represented in Fig. 2. 1 cm displacement represents  $0.98 \times 10^{-8}$  sec.

Run	$E_k$ (MeV)	Signal displacement (cm)	Flight time ( $10^{-8}$ sec)
a	0.5	3.30	3.23
b	1.0	3.14	3.08
c	1.5	2.98	2.92
d	4.5	2.90	2.84
e	15	2.86	2.80

The sweep of the oscilloscope was triggered earlier by another signal transmitted from the beginning of the flight path via a third, shorter cable (Fig. 1). The sweep speed was calibrated

by adding standard cables of known delay time to the cable carrying the timing signal from the beginning of the flight path and then measuring the displacement of the signal on the scope. The delay of these extra cables had been determined prior to the experiment by measuring the resonant frequencies of the cables and checked by measuring the repetition frequency of a self-excited fast pulser with these cables included in the feedback loop (see Appendix).

The signal transit times of the two cables transmitting signals from the ends of the flight path to the oscilloscope were shown to be equal. This was done by using each cable to transmit the timing signal to the oscilloscope from the beginning of the flight path and observing the position of the signal on the oscilloscope face. Since the oscilloscope trigger was always the same, a shift in the position of the signal would indicate a difference in the transit times. This technique established that the differences of the transit times of the signal along the cables was less than  $3 \times 10^{-10}$  sec.

The response times of the pickup circuits at

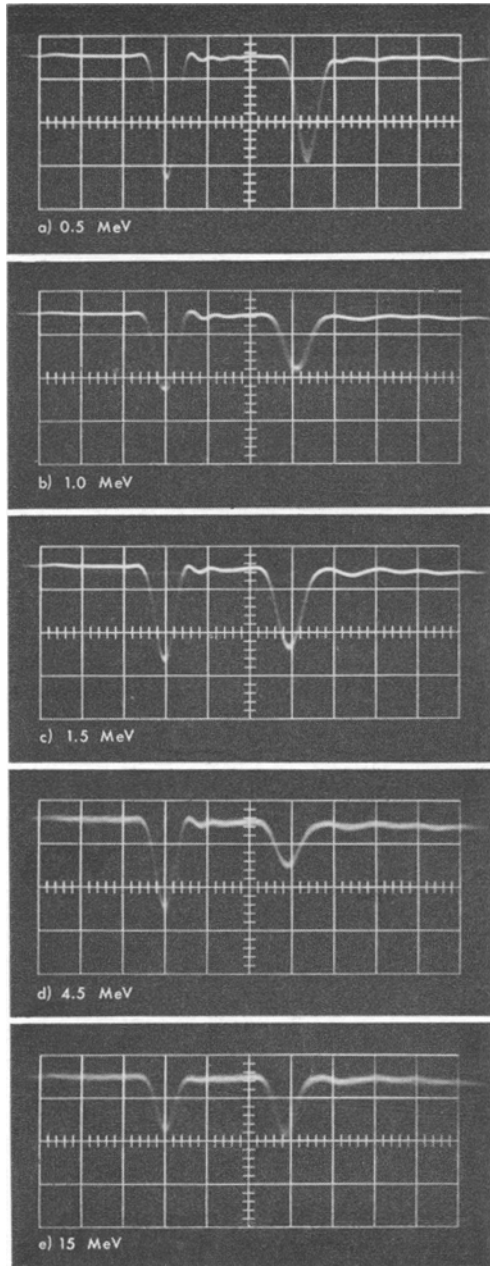


FIG. 2. A sample photograph of the face of the oscilloscope is shown for each of the time-of-flight measurements. The data are presented in Table I. Each large division (1 cm) on the grid of the oscilloscope face represents  $0.98 \times 10^{-8}$  sec.

the beginning and at the end of the flight path were about  $10^{-9}$  sec, and as seen with a fast rise time oscilloscope, the signal from each circuit reproduced approximately the form of the time distribution of charge in each burst. Because the

diameter of the beam emerging from the Van de Graaff was somewhat larger than the diameter of the insulated tube, some of the electrons were stopped by the tube. Therefore, the major component of the signal at the beginning as well as at the end of the flight path was due to collected charge. There is a small component of signal due to a charge induced by the electrons passing through the tube which was not detected with our equipment. With the small time constant used in the pickup circuits this induced component produced two very fast ( $10^{-9}$  sec) spikes of opposite polarity, corresponding to the entry and exit of the burst at the tube. Since the oscilloscope used in the experiment had a rise time of about  $3.5 \times 10^{-9}$  sec, it did not respond significantly to this component. Care was taken to insure that, at the signal level used in the experiments, the timing was not sensitive to the amount of charge caught on the tube. The separation between the two signals on the oscilloscope represents the time of flight of the electron burst with an absolute error of less than  $7 \times 10^{-10}$  sec. However, as indicated above, a change in flight time as small as about  $3 \times 10^{-10}$  sec can be detected.

In Fig. 3, the graph shows the speed squared in units of  $c^2$  as a function of the kinetic energy, and the measured speeds are tabulated. For the energies 0.5, 1.0, and 1.5 MeV the Linac was

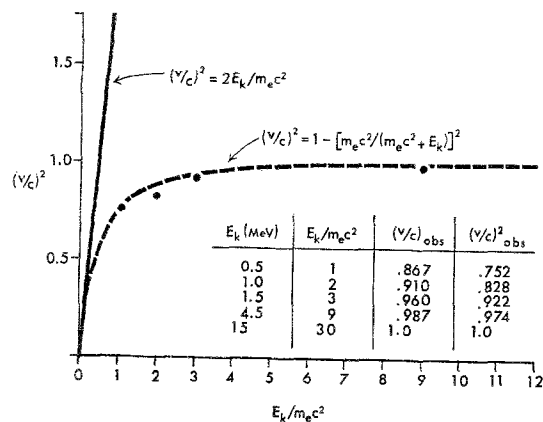


FIG. 3. The solid curve represents the prediction for  $(v/c)^2$  according to Newtonian mechanics,  $(v/c)^2 = 2E_k/mc^2$ . The dashed curve represents the prediction of Special Relativity,  $(v/c)^2 = 1 - [m_0c^2/(m_0c^2 + E_k)]^2$ .  $m_0$  is the rest mass of an electron and  $c$  is the speed of light in a vacuum,  $3 \times 10^8$  M/sec. The solid circles are the data of this experiment. The table presents the observed values of  $v/c$ .

not turned on. To a good approximation it served only as an evacuated metal enclosure, with no accelerating electric fields. The speed measurement for 4.5 MeV was performed with the Van de Graaff set at 1.5 MeV, and with the first section of the Linac (about 1 m long) being used to supply the additional 3 MeV (Fig. 1). In this 4.5 MeV experiment, the speed of the electrons was changing over the first meter of the flight path (since in that distance the energy increased from 1.5 to 4.5 MeV) and thereafter remained constant. Thus, the expression,  $8.4/t$ , defines only an average speed. However, use of the approximation  $v_{4.5} \approx 8.4/t$  is justified by the final experimental result,  $v_{4.5} \approx v_{1.5}$ .

In the 15 MeV run, the electron energy was continually increasing over most of the flight path. However, from the time measurement in Fig. 2(e) one can see that the time of flight is almost the same as that of the 4.5 MeV run, and one can conclude that  $v_{15} \approx v_{4.5}$ ; that is, the speed of the electrons is no longer increasing appreciably. Indeed, the results clearly indicate that as the energy of the electrons is increased, the speed approaches a limiting value,  $3 \times 10^8$  M/sec.

#### ENERGY MEASUREMENTS

Since the purpose of this experiment was to provide simple direct data to test a result of relativity, it is important that the measurements of speed and energy be made as basic and direct as possible.

As discussed above the speeds are determined directly. The kinetic energy, however, is given in terms of the accelerating potential difference ( $V$ ) in the Van de Graaff generator and the electric field ( $\mathcal{E}$ ) in the Linac. This is a well-established procedure that is checked in this laboratory by magnetic deflection methods. It should be noted, however, that use of a voltage or field measurement requires prior knowledge that the force exerted on a charged particle in its direction of motion by an electric field is independent of the speed of the charged particle. Usually, experiments providing this knowledge have been performed at speeds much less than  $c$ , and one might question the validity of this result for high-speed electrons. A magnetic deflection measurement does not relieve the situa-

tion as it involves the relativistic momentum-energy relation as well as the dynamics of motion in a magnetic field. Furthermore, it is clear that the electrons lose energy by inducing currents in the walls as they move down the metal pipe of the Linac. One can show from electromagnetic theory that these losses are negligible, but this is far less satisfactory than experimental proof. It was, therefore, decided to make a direct measurement of the kinetic energy.

When the electrons are stopped in the aluminum disk most of their kinetic energy is lost as heat to the aluminum, resulting in an increase of its temperature. (Only a very small fraction of the electron energy escapes as x radiation.) The total energy input to the disk is proportional to the temperature change of the disk if measurements are made in times short compared to the observed thermal time constant. The energy imparted to the disk was determined by calorimetry, and at the same time the number of electrons carrying this energy was determined by measuring the charge collected on the disk.

The current produced by a thermocouple attached to the disk (Fig. 4) caused a galvanometer to deflect, indicating the temperature rise. Since the time interval during which charge was collected was typically about 7 min, whereas the thermal time constant of the disk was about 50 min, the heat losses during the measurement were negligible. The thermocouple system was cali-

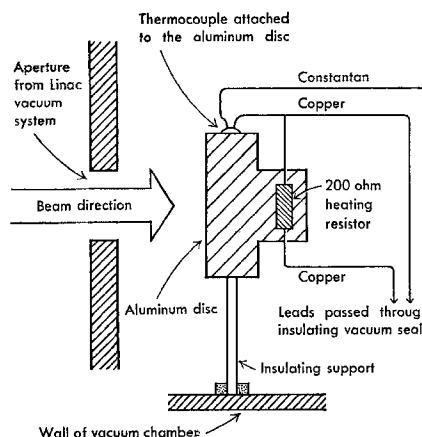


FIG. 4. Schematic diagram of the aluminum disk at the end of the flight path, showing the arrangement of the heating resistor and thermocouple. The signal for the timing runs was taken from the copper wire of the thermocouple.

brated by passing a current through a 200  $\Omega$  resistor also imbedded in the disk. A current of 24 mA flowing for 133 sec caused the galvanometer to deflect 19 divisions. In this way each division of the galvanometer deflection was found to be equivalent to about 0.80 joules of energy absorbed by the aluminum disk.

The beam current flowing into the disk was used to charge a capacitor. A relay was operated to discharge the capacitor whenever its voltage reached a value of about 1 V and each discharge of the capacitor was recorded on a register. The amount of charge represented by each register count was determined by allowing charge to flow on the disk from a 45 V battery in series with a  $5 \times 10^9 \Omega$  resistor. In 15 min, 106 register clicks were observed corresponding to a calibration of  $7.6 \times 10^{-8}$  coulombs per register count.

Energy measurements were performed at the settings of the accelerator used for the 1.5 and 4.5 MeV timing runs. These measurements yielded galvanometer deflections of  $12\frac{1}{2}$  and  $36\frac{1}{2}$  divisions, respectively, when in each run a total charge of about  $6.1 \times 10^{-6}$  coulombs (80 register counts) was collected. From the above calibration the energies indicated by this direct thermal technique were about 1.6 and 4.8 MeV. These agree with the expected energies within the experimental error (about 10%) for all measurements.

#### CONCLUDING REMARKS

In Fig. 3 are plotted the Newtonian relation  $(v/c)^2 = 2E_k/m_e c^2$  and the Einstein relation  $(v/c)^2 = 1 - [m_e c^2 / (m_e c^2 + E_k)]^2$  for electrons. The Newtonian relation, usually adequate for the description of the motion of bodies at very low speeds, is obviously in disagreement with these high-speed data. Equally apparent is the conformity of the experimental data to the Einstein relation with its prediction of a limiting speed.

The experimental data from the calorimetric energy measurements also demonstrate the correctness of our original assumption that each electron moving through a potential difference  $V$  or in an electric field  $\mathcal{E}$  along the direction of its

motion gains an amount of kinetic energy  $E_k = eV$  or  $E_k = \int e \mathcal{E} dx$ , independent of its speed.

#### ACKNOWLEDGMENTS

It is a privilege to acknowledge the collaboration of the late Professor Francis L. Friedman in the conception, planning and all phases of the filming of this experiment. Also, the author wishes to acknowledge gratefully the patient cooperation and help of the staff and graduate students of the photonuclear group of MIT who delayed their own work to allow the filming of this experiment. The assistance of Stanley Kowalski was essential to the performance of several parts of the experiment. The author is very appreciative of the help of Dr. James Strickland and Professor N. H. Frank in the script writing and the preparation of this paper; and the helpful criticism of Professor A. P. French and his other colleagues at the Science Teaching Center.

#### APPENDIX

The resonance frequencies,  $f_n$ , of the cable determine the signal transit speed  $v$ , through the relation  $v = \lambda_n f_n$ , where  $\lambda_n$  is the wavelength corresponding to  $f_n$ . For example, if one of the cable terminations is a short circuit and the other open, then  $\lambda_n = 4L/n$ , where  $n$  is an odd integer and  $L$  is the cable length. The  $f_n$  may be measured with the appropriate use of an oscillator and detector. The transit time of a fast pulse on a cable can also be measured directly by using a self-excited pulser. The output signal of a fast pulser is used to start the same pulser after a suitable delay time. This is achieved by passing the output through a delay cable to the input of the pulser. The delay of the cable is sufficient to allow the pulser to settle into its quiescent state. In this mode the pulser output repeats at some rate  $f_0$ . If the length of the delay cable is now increased by the addition of a cable of unknown transit time the repetition frequency decreases to  $f_1$ . The transit time of the signal along the added length is then given by  $t = 1/f_0 - 1/f_1$ .