Máme rovnici: $1/T = a + b*\ln(R) + c*(\ln(R))^3$

chceme najít konstanty a, b, c

mají vyjít a = 1, $4*10^{-3}$, b = 2, $37*10^{-4}$, c = 9, $9*10^{-8}$

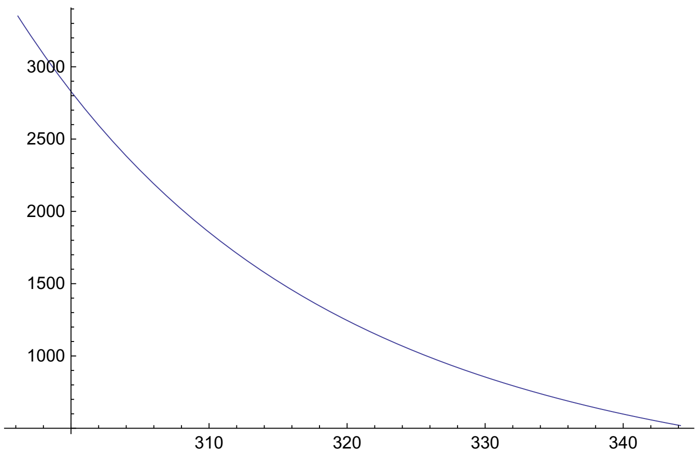
zdroj Wikipedie Thermistor

upravená výchozí rovnice pro odpor - z Wikipedie (možná jsem tam udělal nějakou chybu)

$$R := e^{\left(\sqrt{\left(\frac{b}{3*c}\right)^3+\left(0.5*\frac{1}{c}*\left(a-\frac{1}{T}\right)\right)^2}-0.5*\frac{1}{c}*\left(a-\frac{1}{T}\right)\right)^{\frac{1}{3}}-\left(\sqrt{\left(\frac{b}{3*c}\right)^3+\left(0.5*\frac{1}{c}*\left(a-\frac{1}{T}\right)\right)^2}+0.5*\frac{1}{c}*\square^{\left(a-\frac{1}{T}\right)}\right)^{\frac{1}{3}}} \;/.$$

$$\left\{a \to 1.4*10^{-3},\ b \to 2.37*10^{-4},\ c \to 9.9*10^{-8}\right\}$$

```
Plot[R, {T, 296.15, 344.15}]
```

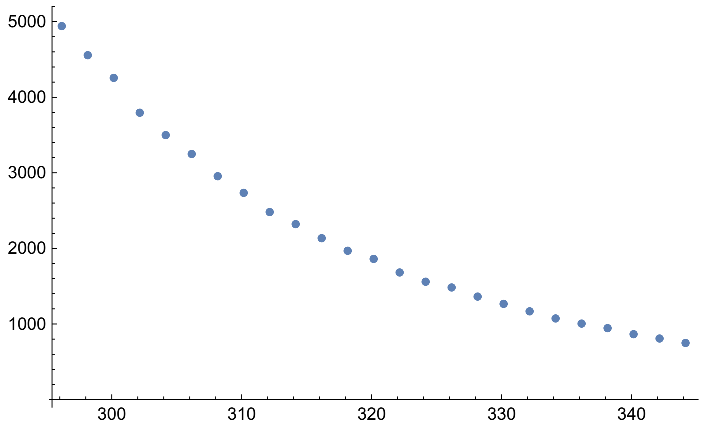



```
data = {{296.15, 4941}, {298.15, 4556}, {300.15, 4256}, {302.15, 3795},
  {304.15, 3499}, {306.15, 3249}, {308.15, 2955}, {310.15, 2735}, {312.15, 2481},
  {314.15, 2321}, {316.15, 2135}, {318.15, 1968.7}, {320.15, 1861},
  {322.15, 1681.7}, {324.15, 1559.2}, {326.15, 1483.2}, {328.15, 1362.5},
  {330.15, 1266.3}, {332.15, 1167.5}, {334.15, 1073.4}, {336.15, 1005.2},
  {338.15, 945.1}, {340.15, 865}, {342.15, 807.9}, {344.15, 748.8}}
```

```
{{296.15, 4941}, {298.15, 4556}, {300.15, 4256}, {302.15, 3795},
 {304.15, 3499}, {306.15, 3249}, {308.15, 2955}, {310.15, 2735}, {312.15, 2481},
 {314.15, 2321}, {316.15, 2135}, {318.15, 1968.7}, {320.15, 1861},
 {322.15, 1681.7}, {324.15, 1559.2}, {326.15, 1483.2}, {328.15, 1362.5},
 {330.15, 1266.3}, {332.15, 1167.5}, {334.15, 1073.4}, {336.15, 1005.2},
 {338.15, 945.1}, {340.15, 865}, {342.15, 807.9}, {344.15, 748.8}}
```

naměřená data, první je teplota v K, druhé je odpor v ohmnech

```
ListPlot[data]
```



potřeboval bych nafitovat konstanty a, b, c a graficky znázornit - asi by bylo nejlepší dát nafitované konstaty do R := $e^{\left(\sqrt{\left(\frac{b}{3*c}\right)^3+\left(0.5*\frac{1}{c}*\left(a-\frac{1}{T}\right)\right)^2}-0.5*\frac{1}{c}*\left(a-\frac{1}{T}\right)\right)^{\frac{1}{3}}-\left(\sqrt{\left(\frac{b}{3*c}\right)^3+\left(0.5*\frac{1}{c}*\left(a-\frac{1}{T}\right)\right)^2}+0.5*\frac{1}{c}*\left(a-\frac{1}{T}\right)\right)^{\frac{1}{3}}}$

a zobrazit do 1 grafu “teoretický” graf a “reálný” graf - jeden pro dané konstaty, druhý pro nafitované

```
(* Přesupořádání hodnot *)
hodnoty = Partition[Riffle[data[[All, 2]], 1 / data[[All, 1]]], 2]
(* hodnoty1=Partition[Riffle[data[[All,2]],data[[All,1]]],2]*)
```

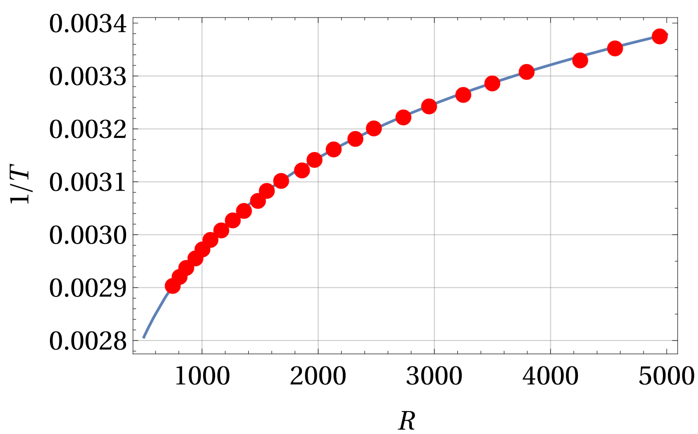
```
{{4941, 0.00337667}, {4556, 0.00335402}, {4256, 0.00333167}, {3795, 0.00330961},
 {3499, 0.00328785}, {3249, 0.00326637}, {2955, 0.00324517}, {2735, 0.00322425},
 {2481, 0.00320359}, {2321, 0.00318319}, {2135, 0.00316306}, {1968.7, 0.00314317},
 {1861, 0.00312354}, {1681.7, 0.00310414}, {1559.2, 0.00308499},
 {1483.2, 0.00306607}, {1362.5, 0.00304739}, {1266.3, 0.00302893},
 {1167.5, 0.00301069}, {1073.4, 0.00299267}, {1005.2, 0.00297486},
 {945.1, 0.00295727}, {865, 0.00293988}, {807.9, 0.00292269}, {748.8, 0.00290571}}
```

```
(* První způsob *)
nlinmodel = NonlinearModelFit[hodnoty, a + b * Log[x] + c * Log[x] ^3, {a, b, c}, x];
{nlinmodel["BestFit"], nlinmodel["ParameterTable"],
 nlinmodel["RSquared"], Sqrt[nlinmodel["RSquared"]]}
Show[{Plot[nlinmodel["BestFit"] /. x → R, {R, 500, 5000},
   PlotStyle → {Thickness[0.005]}, FrameLabel → {"R", "1/T"}, BaseStyle →
    {FontFamily → "Times", FontSize → 14}, GridLines → Automatic, Frame → True],
  ListPlot[hodnoty, PlotMarkers → {Automatic, Medium}, PlotStyle → {Red}]}]
```

{0.00142187 + 0.000214882 Log[x] + 2.04955 × 10^{-7} Log[x]^3,

	Estimate	Standard Error	t-Statistic	P-Value
a	0.00142187	0.0000665654	21.3604	3.35294 × 10^{-16}
b	0.000214882	0.0000132632	16.2014	1.03218 × 10^{-13}
c	2.04955 × 10^{-7}	7.69773 × 10^{-8}	2.66254	0.0142228

, 0.999999, 1.}

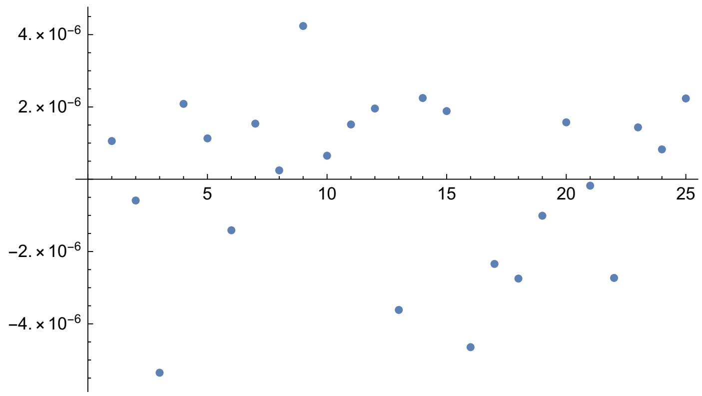



```
(* parametry modelu *)
nlinmodel["ParameterConfidenceIntervals"]
nlinmodel["ParameterConfidenceIntervals", ConfidenceLevel → .99]
resids = nlinmodel["FitResiduals"]
ListPlot[resids]
nlinmodel["SinglePredictionConfidenceIntervalTable"]
```

{{0.00128382, 0.00155991}, {0.000187376, 0.000242388}, {4.5314 × 10^{-8}, 3.64596 × 10^{-7}}}

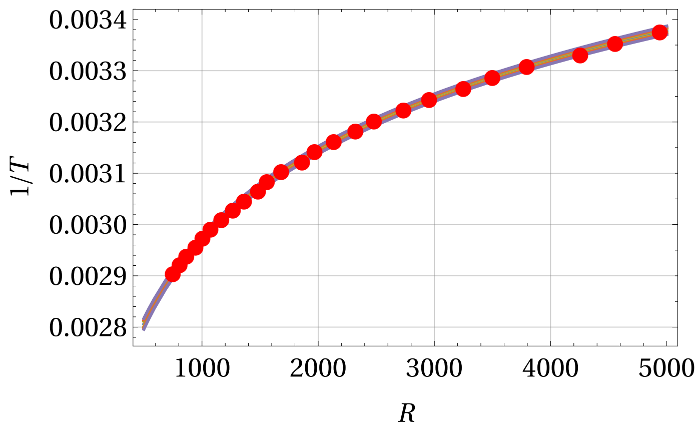
{{0.00123423, 0.0016095}, {0.000177496, 0.000252268}, {-1.20251 × 10^{-8}, 4.21936 × 10^{-7}}}

{1.05718 × 10^{-6}, -5.87809 × 10^{-7}, -5.35174 × 10^{-6}, 2.08519 × 10^{-6}, 1.13021 × 10^{-6},
 -1.41164 × 10^{-6}, 1.53836 × 10^{-6}, 2.45245 × 10^{-7}, 4.23882 × 10^{-6}, 6.51326 × 10^{-7},
 1.51502 × 10^{-6}, 1.95595 × 10^{-6}, -3.61503 × 10^{-6}, 2.2462 × 10^{-6}, 1.88521 × 10^{-6},
 -4.64509 × 10^{-6}, -2.34231 × 10^{-6}, -2.74755 × 10^{-6}, -1.0098 × 10^{-6}, 1.57426 × 10^{-6},
 -1.78189 × 10^{-7}, -2.72999 × 10^{-6}, 1.43534 × 10^{-6}, 8.26211 × 10^{-7}, 2.23463 × 10^{-6}}



Observed	Predicted	Standard Error	Confidence Interval
0.00337667	0.00337561	2.90202×10^{-6}	{0.00336959, 0.00338163}
0.00335402	0.0033546	2.79607×10^{-6}	{0.00334881, 0.0033604}
0.00333167	0.00333702	2.72993×10^{-6}	{0.00333136, 0.00334268}
0.00330961	0.00330753	2.65788×10^{-6}	{0.00330202, 0.00331304}
0.00328785	0.00328672	2.63032×10^{-6}	{0.00328127, 0.00329218}
0.00326637	0.00326778	2.61753×10^{-6}	{0.00326236, 0.00327321}
0.00324517	0.00324363	2.61293×10^{-6}	{0.00323822, 0.00324905}
0.00322425	0.003224	2.61508×10^{-6}	{0.00321858, 0.00322942}
0.00320359	0.00319935	2.62095×10^{-6}	{0.00319391, 0.00320478}
0.00318319	0.00318254	2.62507×10^{-6}	{0.0031771, 0.00318799}
0.00316306	0.00316154	2.62876×10^{-6}	{0.00315609, 0.00316699}
0.00314317	0.00314122	2.62983×10^{-6}	{0.00313576, 0.00314667}
0.00312354	0.00312715	2.6289×10^{-6}	{0.0031217, 0.0031326}
0.00310414	0.0031019	2.62399×10^{-6}	{0.00309646, 0.00310734}
0.00308499	0.00308311	2.61832×10^{-6}	{0.00307768, 0.00308854}
0.00306607	0.00307072	2.61419×10^{-6}	{0.0030653, 0.00307614}
0.00304739	0.00304973	2.60782×10^{-6}	{0.00304432, 0.00305514}
0.00302893	0.00303167	2.60469×10^{-6}	{0.00302627, 0.00303708}
0.00301069	0.0030117	2.60634×10^{-6}	{0.00300629, 0.0030171}
0.00299267	0.00299109	2.61703×10^{-6}	{0.00298567, 0.00299652}
0.00297486	0.00297504	2.63423×10^{-6}	{0.00296958, 0.0029805}
0.00295727	0.00296	2.65938×10^{-6}	{0.00295448, 0.00296551}
0.00293988	0.00293844	2.71413×10^{-6}	{0.00293282, 0.00294407}
0.00292269	0.00292187	2.77394×10^{-6}	{0.00291612, 0.00292762}
0.00290571	0.00290348	2.861×10^{-6}	{0.00289754, 0.00290941}

```
(*vykreslení s konfidenčními intervaly *)
{bands80[x_], bands90[x_], bands95[x_], bands99[x_]} =
  Table[nlinmodel["SinglePredictionBands", ConfidenceLevel → cl],
   {cl, {.8, .9, .95, .99}}];
Show[{Plot[{nlinmodel[x], bands80[x], bands90[x], bands95[x], bands99[x]},
   {x, 500, 5000}, Filling → {2 → {1}, 3 → {2}, 4 → {3}, 5 → {4}},
   FrameLabel → {"R", "1/T"}, BaseStyle → {FontFamily → "Times", FontSize → 14},
   GridLines → Automatic, Frame → True],
  ListPlot[hodnoty, PlotMarkers → {Automatic, Medium}, PlotStyle → {Red}]}]
```



```
(* Druhý způsob *)
model = LinearModelFit[hodnoty, {x, Log[x], Log[x]^3}, x]
model["BestFit"]
{model["ParameterTable"], model["RSquared"], Sqrt[nlinmodel["RSquared"]]}
Show[{Plot[model["BestFit"], {x, 500, 5000}, PlotStyle → {Thickness[0.005]},
   FrameLabel → {"R", "1/T"}, BaseStyle → {FontFamily → "Times", FontSize → 14},
   Frame → True, GridLines → Automatic],
  ListPlot[hodnoty, PlotMarkers → {Automatic, Medium}, PlotStyle → {Red}]}]
```

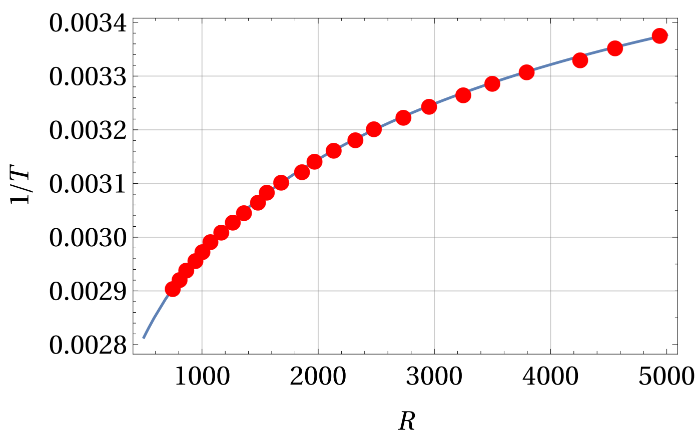
FittedModel[$0.0019203 - 1.96282\times10^{-8}\,x + 0.000102965\,\text{Log}[x] + 1.09651\times10^{-6}\,\text{Log}[x]^3$]

$0.0019203 - 1.96282\times10^{-8}\,x + 0.000102965\,\text{Log}[x] + 1.09651\times10^{-6}\,\text{Log}[x]^3$

{

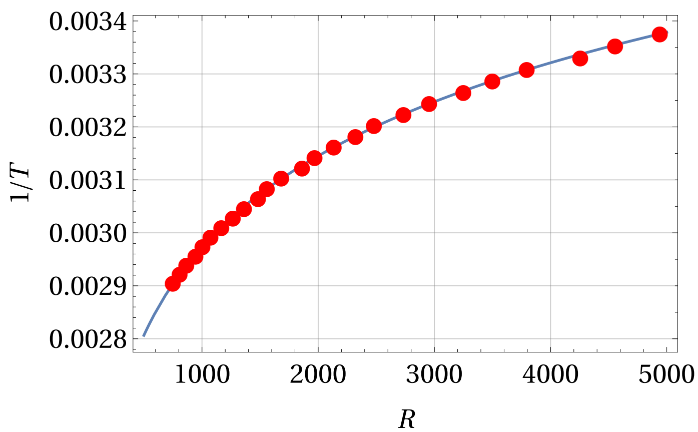
	Estimate	Standard Error	t-Statistic	P-Value
1	0.0019203	0.000300806	6.38384	2.50032×10^{-6}
x	-1.96282×10^{-8}	1.15754×10^{-8}	-1.69569	0.104722
Log[x]	0.000102965	0.0000672173	1.53183	0.140491
Log[x]^3	1.09651×10^{-6}	5.30944×10^{-7}	2.06521	0.0514668

, 0.999754, 1.}



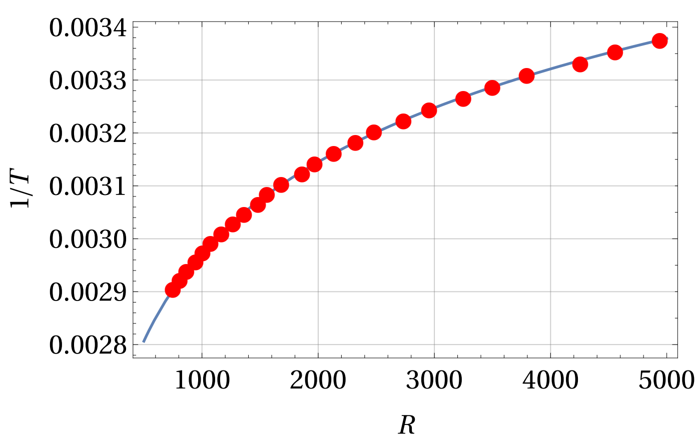
```
(* Třetí způsob, bez vyčíslení korelace a chyby *)
nlinmodela = FindFit[hodnoty, a1 + b1 * Log[x] + c1 * Log[x] ^ 3, {a1, b1, c1}, x]
Show[{Plot[a1 + b1 * Log[R] + c1 * Log[R] ^ 3 /. nlinmodela, {R, 500, 5000},
    PlotStyle → {Thickness[0.005]}, FrameLabel → {"R", "1/T"}, BaseStyle →
     {FontFamily → "Times", FontSize → 14}, GridLines → Automatic, Frame → True],
  ListPlot[hodnoty, PlotMarkers → {Automatic, Medium}, PlotStyle → {Red}]}]
```

{a1 → 0.00142187, b1 → 0.000214882, c1 → 2.04955×10^{-7}}




```
(* Čtvrtý způsob, bez vyčíslení korelace a chyby *)
nlinmodelb = Fit[hodnoty, {1, Log[x], Log[x] ^ 3}, x]
Show[{Plot[nlinmodelb /. x → R, {R, 500, 5000}, PlotStyle → {Thickness[0.005]},
    FrameLabel → {"R", "1/T"}, BaseStyle → {FontFamily → "Times", FontSize → 14},
    GridLines → Automatic, Frame → True],
  ListPlot[hodnoty, PlotMarkers → {Automatic, Medium}, PlotStyle → {Red}]}]
```

$0.00142187 + 0.000214882 \, \mathrm{Log}[x] + 2.04955 \times 10^{-7} \, \mathrm{Log}[x]^3$

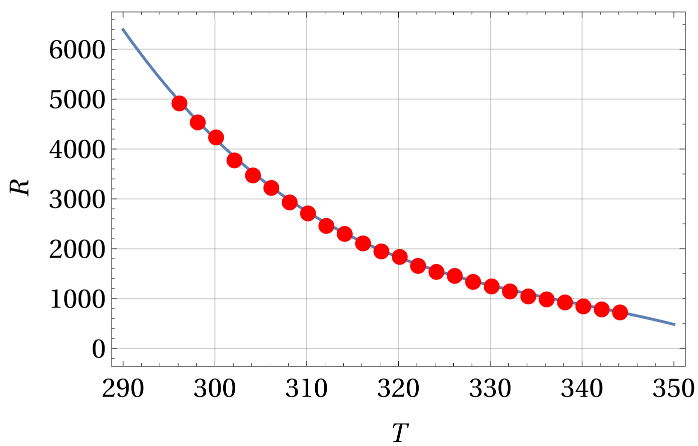


```
(* Obrácená závislost aprocimací polynomem *)
nlinmodelinv = NonlinearModelFit[data, a + b * x + c * x^2 + d * x^3, {a, b, c, d}, x];
{nlinmodelinv["BestFit"], nlinmodelinv["ParameterTable"],
 nlinmodelinv["RSquared"], Sqrt[nlinmodelinv["RSquared"]]}
Show[{Plot[nlinmodelinv["BestFit"] /. x → T, {T, 290, 350},
   PlotStyle → {Thickness[0.005]}, FrameLabel → {"T", "R"}, BaseStyle →
    {FontFamily → "Times", FontSize → 14}, GridLines → Automatic, Frame → True],
  ListPlot[data, PlotMarkers → {Automatic, Medium}, PlotStyle → {Red}]}]
```

$\{1.20816 \times 10^6 - 10595.6\, x + 31.1026\, x^2 - 0.0305372\, x^3,$

	Estimate	Standard Error	t-Statistic	P-Value
a	1.20816×10^6	72602.8	16.6406	1.43057×10^{-13}
b	−10595.6	681.697	−15.543	5.40915×10^{-13}
c	31.1026	2.13135	14.5929	1.82672×10^{-12}
d	−0.0305372	0.00221892	−13.7622	5.59484×10^{-12}

$, 0.99991, 0.999955\}$

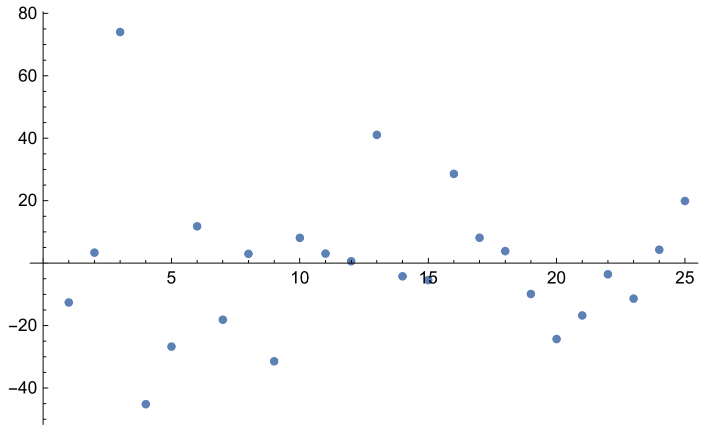


```
(* parametry modelu *)
nlinmodelinv["ParameterConfidenceIntervals"]
nlinmodelinv["ParameterConfidenceIntervals", ConfidenceLevel → .99]
residsinv = nlinmodelinv["FitResiduals"]
ListPlot[residsinv]
nlinmodelinv["SinglePredictionConfidenceIntervalTable"]
```

{{1.05717 × 10^6, 1.35914 × 10^6}, {-12 013.3, -9177.93},
{26.6702, 35.535}, {-0.0351517, -0.0259227}}

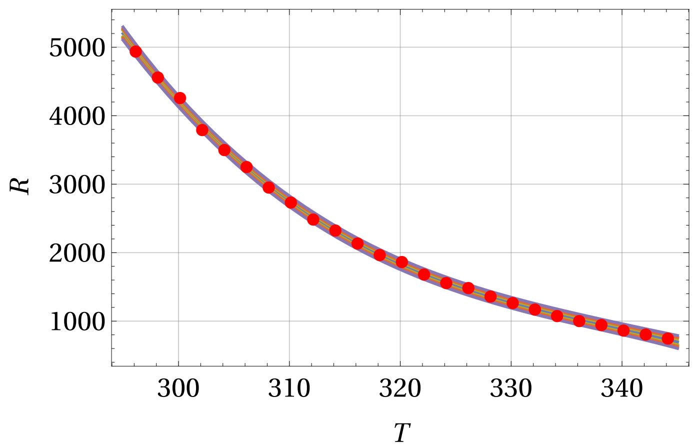
{{1.00259 × 10^6, 1.41372 × 10^6}, {-12 525.7, -8665.46},
{25.068, 37.1372}, {-0.0368198, -0.0242547}}

{-12.6077, 3.35718, 74.0134, -45.1733, -26.7372, 11.7877, -18.133, 2.96658, -31.4478,
8.08962, 3.04466, 0.583099, 41.0707, -4.22668, -5.44332, 28.5866, 8.12882,
3.84917, -9.88656, -24.3126, -16.7632, -3.57246, -11.3747, 4.2959, 19.9051}



Observed	Predicted	Standard Error	Confidence Interval
4941	4953.61	31.5379	{4888.02, 5019.19}
4556	4552.64	29.1447	{4492.03, 4613.25}
4256	4181.99	27.9327	{4123.9, 4240.08}
3795	3840.17	27.4721	{3783.04, 3897.3}
3499	3525.74	27.3997	{3468.76, 3582.72}
3249	3237.21	27.4648	{3180.1, 3294.33}
2955	2973.13	27.5248	{2915.89, 3030.37}
2735	2732.03	27.5194	{2674.8, 2789.26}
2481	2512.45	27.4425	{2455.38, 2569.52}
2321	2312.91	27.3198	{2256.1, 2369.73}
2135	2131.96	27.1908	{2075.41, 2188.5}
1968.7	1968.12	27.0945	{1911.77, 2024.46}
1861	1819.93	27.0591	{1763.66, 1876.2}
1681.7	1685.93	27.0945	{1629.58, 1742.27}
1559.2	1564.64	27.1908	{1508.1, 1621.19}
1483.2	1454.61	27.3199	{1397.8, 1511.43}
1362.5	1354.37	27.4426	{1297.3, 1411.44}
1266.3	1262.45	27.5195	{1205.22, 1319.68}
1167.5	1177.39	27.5249	{1120.15, 1234.63}
1073.4	1097.71	27.4648	{1040.6, 1154.83}
1005.2	1021.96	27.3997	{964.982, 1078.94}
945.1	948.672	27.4721	{891.541, 1005.8}
865	876.375	27.9327	{818.286, 934.464}
807.9	803.604	29.1447	{742.994, 864.214}
748.8	728.895	31.5378	{663.308, 794.481}

```
(*vykreslení s konfidenčními intervaly *)
{bands80[x_], bands90[x_], bands95[x_], bands99[x_]} =
  Table[nlinmodelinv["SinglePredictionBands", ConfidenceLevel → cl],
   {cl, {.8, .9, .95, .99}}];
Show[{Plot[{nlinmodelinv[x], bands80[x], bands90[x], bands95[x], bands99[x]},
    {x, 295, 345}, Filling → {2 → {1}, 3 → {2}, 4 → {3}, 5 → {4}},
    FrameLabel → {"T", "R"}, BaseStyle → {FontFamily → "Times", FontSize → 14},
    GridLines → Automatic, Frame → True],
  ListPlot[data, PlotMarkers → {Automatic, Small}, PlotStyle → {Red}]}]
```



```
(* Obrácená závislost aprocimací polynomem *)
nlinmodelinva =
  NonlinearModelFit[data, Exp[a + b / x + c / x^2 + d / x^3], {a, b, c, d}, x];
{nlinmodelinva["BestFit"], nlinmodelinva["ParameterTable"],
 nlinmodelinva["RSquared"], Sqrt[nlinmodelinva["RSquared"]]}
Show[{Plot[nlinmodelinva["BestFit"] /. x → T, {T, 290, 350},
    PlotStyle → {Thickness[0.005]}, FrameLabel → {"T", "R"}, BaseStyle →
     {FontFamily → "Times", FontSize → 14}, GridLines → Automatic, Frame → True],
  ListPlot[data, PlotMarkers → {Automatic, Medium}, PlotStyle → {Red}]}]
```

NonlinearModelFit: The step size in the search has become less than the tolerance prescribed by the PrecisionGoal option, but the gradient is larger than the tolerance specified by the AccuracyGoal option. There is a possibility that the method has stalled at a point that is not a local minimum.

$\left\{e^{-2.25663-\frac{1.46282\times 10^{8}}{x^{3}}+\frac{1.19362\times 10^{6}}{x^{2}}+\frac{826.423}{x}},\right.$

	Estimate	Standard Error	t-Statistic	P-Value
a	−2.25663	38.964	−0.0579157	0.954363
b	826.423	36 825.3	0.0224417	0.982307
c	1.19362×10^{6}	1.15904×10^{7}	0.102984	0.918953
d	-1.46282×10^{8}	1.21486×10^{9}	−0.120411	0.905302

$, 0.999909, 0.999955\}$