Quantum theory of quantum-well polaritons in semiconductor microcavities

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The polariton Hamiltonian for a quantum well embedded in a cavity of finite width is derived. The exciton-light interaction is formulated in second quantization and the photon self-interaction term $\mathbf{A}^2$ is included. The importance of the $\mathbf{A}^2$ term turns out to be relevant in the case of large values of the cavity width. By solving the equation of motion the dispersion relations for upper and lower polaritons are found. A comparison is made with existing measurements of polariton absorption peaks. The limit of infinite cavity width is carried out and the known results for a quantum well in an infinite medium are recovered.

I. INTRODUCTION

Polaritons in quantum wells (QW's) have been the subject of many interesting studies in the last few years. In a QW the breaking of the translational symmetry along the **z** axis completely modifies the selection rules of the interaction between excitons and photons with respect to the bulk case. A QW exciton with a given in-plane wave vector couples to a continuum of photon modes. The in-plane component of the photon wave vector is equal to that of the exciton while the **z** component can assume all the possible values. Because of this peculiarity of the coupling, one of the polariton modes turns out to be a resonant mode with a radiative linewidth. Polaritons in QW's have been the object of investigation in the framework of both classical[1] and quantum[2,3] theory. Recently the possibility of growing optical microcavities led to the study of the effect of light confinement on QW polaritons. A semiconductor microcavity is a planar Fabry-Pérot whose mirrors are multilayer structures built with alternating layers of two different refraction indexes and the same thickness $\lambda/4$, where λ is the resonance wavelength of the Fabry-Pérot. A structure of this kind, called distributed Bragg reflector (DBR), presents a very high reflectivity in a given frequency interval around the resonance. The main difference between a DBR and a common metallic mirror is the following: The DBR reflection coefficient for the electric field has a phase equal to zero at the resonance, whereas for a metallic mirror this phase is equal to π. In other words standing waves inside the DBR Fabry-Pérot present antinodes instead of nodes at the cavity boundaries. In what follows we will adopt the boundary conditions appropriate to describe this feature. Another characteristic is the frequency dependence of the reflection coefficient of a DBR. Since we are mainly interested in the behavior of polaritons near the resonance, we disregard this dependence.

As shown experimentally,[4,5] this system shows features, such as vacuum field Rabi oscillations, analogous to those of atoms enclosed in cavities.[6–9] The vacuum field Rabi splitting in this case is interpreted as one aspect of the polariton effect. Other features, like exciton center of mass quantization, can be observed when the whole body of the cavity is used as the optical active medium.[10,11] The main difference between atom and QW experiments is the following: Light modes in semiconductor microcavities are confined only in one dimension, whereas in the other two dimensions the microcavity has full translational symmetry. As already mentioned, interaction will be possible only between excitons and cavity modes with the same in-plane wave vector. Because of this selection rule, excitons in an ideal lossless cavity are interacting only with a discrete set of cavity modes. This reproduces the same situation as in experiments with atoms, where particular cavity geometries had to be used for this purpose. The study of this kind of confined system is very interesting, both from the point of view of the possible technological developments, and from that of theoretical and experimental investigation of some peculiar aspects of quantum mechanics, including controlled spontaneous emission, and quantum nondemolition measurements. These issues are part of the theory known as cavity quantum electrodynamics (CQED).[12]

In this paper we present a theoretical investigation of the polaritons in a QW enclosed in a microcavity with perfectly reflecting walls, implying only discrete cavity modes. This idealization is justified because of the high finesse achieved in the experimental setup.[5] The whole analysis is carried out in the framework of second quantization and the photon self-interaction term $\mathbf{A}^2$ is included in the Hamiltonian. In Sec. II we derive the form of the Hamiltonian from a microscopical model including the main features of the exciton system. Without losing any generality, we will consider only a single polarization for light modes, namely, the polarization orthogonal to the **z** axis, as well as a single exciton mode. Moreover, the cavity here will be considered as lossless, the cavity modes being discrete modes with zero linewidth. The

microscopical model considered here for the exciton contains only its essential features. The exciton confinement function is taken to be zero outside the QW and constant inside. This assumption does not bring any significant change in the dispersion relations when the cavity width is comparable with the exciton wavelength or larger. Furthermore, the exciton dispersion is neglected. In Sec. III we will derive the dispersion relations for the polariton modes. A comparison with existing experimental data is carried out and a satisfactory agreement is found. The importance of the $\mathbf{A}^2$ term, neglected in Ref. 13, is discussed in detail. In a QW there exist two different types of polariton modes which are called resonance and surface polaritons.[1] However, when the QW is confined in an ideal lossless cavity with only discrete light modes, the resonant behavior characterizing resonance polaritons in a free QW is not present, both polariton modes having zero radiative linewidth. Therefore these modes will be denoted as upper and lower polaritons, respectively, in analogy with bulk polaritons. Carrying out the limit of an infinite cavity the behavior of the QW in an infinite dielectric medium[1] can be recovered. The method to perform such a limiting process is devised in Sec. IV. In Sec. V a discussion of the results and of their implications is presented.

II. POLARITON HAMILTONIAN

We consider the system of a QW of thickness L placed at the center of a slab of dielectric material of thickness L' which constitutes the microcavity. We stress on the fact that the whole system keeps the translational invariance along the plane orthogonal to the $\mathbf{z}$ direction. Furthermore, we disregard any dispersive behavior of the microcavity, taking into account only its dielectric constant ϵ_0. In the approximation in which only one conduction and one valence band are considered and the spin is neglected, an exciton state with an in-plane wave vector $\mathbf{k}$ is given by

$$|\Phi_{\mathbf{k}}\rangle = \sum_{\mathbf{k}_T} A(\mathbf{k}_T, \mathbf{k}) c^{\dagger}_{\mathbf{k}_T + \frac{m_e}{M}\mathbf{k}} d^{\dagger}_{-\mathbf{k}_T + \frac{m_h}{M}\mathbf{k}} |0\rangle, \tag{1}$$

where $M = m_e + m_h$ and $c^{\dagger}$ is the creation operator for a conduction electron, while $d^{\dagger}$ is the creation operator for a hole in the valence band. $A(\mathbf{k}_T, \mathbf{k})$ is the Fourier transform of the exciton envelope function $F_{\mathbf{k}}(\boldsymbol{\rho})$ where $\boldsymbol{\rho}$ is the in-plane displacement vector. In particular we have

$$\frac{1}{\sqrt{S}} \sum_{\mathbf{k}_T} A(\mathbf{k}_T, \mathbf{k}) = F_{\mathbf{k}}(0), \tag{2}$$

where S is the normalization surface.

The states $c^{\dagger}_{\mathbf{k}_1} d^{\dagger}_{\mathbf{k}_2} |0\rangle$ are given by Slater determinants where the conduction wave function $\phi^{(c)}_{\mathbf{k}_1}(\mathbf{x})$ replaces the valence wave function $\phi^{(v)}_{-\mathbf{k}_2}(\mathbf{x})$ in the Slater determinant corresponding to the full valence band. Furthermore, the electron wave functions can be written in the envelope function approximation as

$$\phi^{(v,c)}_{\mathbf{k}}(\mathbf{x}) = \frac{\exp(i\mathbf{k}\cdot\boldsymbol{\rho})}{\sqrt{S}} f_{(v,c)}(z) u_{(v,c)}(\mathbf{x}), \tag{3}$$

where $f_{(v,c)}(z)$ is the envelope function in the $\mathbf{z}$ direction and $u_{(v,c)}(\mathbf{x})$ is a Bloch function taken at the Γ point in the Brillouin zone.

Let us now consider the interaction Hamiltonian in the form

$$H_I = -\frac{e}{mc} \sum_i \mathbf{A}(\mathbf{x}_i) \cdot \mathbf{p}_i + \frac{e^2}{2mc^2} \sum_i \mathbf{A}^2(\mathbf{x}_i), \tag{4}$$

where the sum is over the electrons of the system and $\mathbf{x}_i$ is the three-dimensional displacement vector, namely, $\mathbf{x}_i = (\boldsymbol{\rho}_i, z_i)$. We begin by writing in second quantization form the first term on the right hand side of (4) which we will call $H^{(1)}_I$. The most general form for this Hamiltonian, when a nonlocal potential is present in the electron one-particle Hamiltonian, is[14]

$$H^{(1)}_I = -\frac{e}{c} \sum_i \mathbf{A}(\mathbf{x}_i) \cdot \mathbf{v}_i, \tag{5}$$

where $\mathbf{v}_i$ is the ith electron velocity and the relation

$$\mathbf{v}_i = \frac{1}{i\hbar}[\mathbf{x}_i, H_{\mathrm{exc}}] \tag{6}$$

is valid. Here H_{exc} denotes the exciton Hamiltonian.

We consider only light polarized orthogonally to the $\mathbf{z}$ direction and we let the electric field amplitude be maximum at the cavity walls. This choice of boundary conditions, as already mentioned, allows to compare our results with the available experimental ones. Furthermore, we consider only even modes because, due to the parity selection rule, these are the only ones which couple with the exciton ground state. In this way the z dependence of the electromagnetic modes will be given by the factor $\cos(q_z z)$ where $q_z = 2n\pi/L'$ and n takes any integer value in the range $[-\infty, +\infty]$.

The expression for the vector potential $\mathbf{A}$ in second quantization form will then be

$$\mathbf{A}(\mathbf{x}) = \sum_{\mathbf{q},q_z} \sqrt{\frac{2\pi\hbar v}{SL'(q^2+q_z^2)^{\frac{1}{2}}}} [A^1_{\mathbf{q},q_z} \exp(i\mathbf{q}\cdot\boldsymbol{\rho}) + A^{1\dagger}_{\mathbf{q},q_z} \exp(-i\mathbf{q}\cdot\boldsymbol{\rho})] \cos(q_z z) \boldsymbol{\epsilon}_{\mathbf{q},q_z}. \tag{7}$$

In this expresion $\mathbf{q}$ is the in-plane wave vector for the electromagnetic field and $v = c/\sqrt{\epsilon_0}$. In order to write the Hamiltonian (5) in second quantization form we define $A^2_{\mathbf{k}}$, $A^{2\dagger}_{\mathbf{k}}$ as the annihilation and creation operators for an exciton with given in–plane wave vector $\mathbf{k}$, respectively. We can write the Hamiltonian as

$$H^{(1)}_I = -\sum_{\mathbf{k}} \left(\langle 0| \frac{e}{c} \sum_i \mathbf{A}(\mathbf{x}_i) \cdot \mathbf{v}_i |\Phi_{\mathbf{k}}\rangle A^2_{\mathbf{k}} + \quad \mathrm{H.c.} \right). \tag{8}$$

The operators $A^2_{\mathbf{k}}$, $A^{2\dagger}_{\mathbf{k}}$ obey Bose commutation rules which as is well known implies a low excitation regime.

At this point it is enough to substitute Eqs. (6) and (7) into (8) and to write the exciton states as linear combinations of Slater determinants using (3). Introducing the dipole approximation and making the further assumption that the confinement function $\rho(z)$, defined as $\rho(z) = f_v^*(z) f_c(z)$, is approximately constant over a Brillouin zone, after somewhat lengthy calculations it is a straightforward matter to obtain the final form for $H_I^{(1)}$:

$$H_I^{(1)} = \sum_{\mathbf{q},q_z} iC_{\mathbf{q},q_z}(A^2_{-\mathbf{q}} - A^{2\dagger}_{\mathbf{q}})(A^1_{\mathbf{q},q_z} + A^{1\dagger}_{-\mathbf{q},-q_z}), \quad (9)$$

with

$$C_{\mathbf{q},q_z} = \frac{\omega_{\mathbf{q}}}{c}\sqrt{\frac{2\pi\hbar v}{L'}}\frac{1}{(q^2+q_z^2)^{\frac{1}{4}}}F_{\mathbf{q}}(0)\mu_{cv}(\mathbf{q},q_z)I(q_z). \quad (10)$$

In this expression $\omega_{\mathbf{q}} = E_{\mathbf{q}}/\hbar$ is the exciton dispersion, $\mu_{cv}(\mathbf{q},q_z) = e\langle v|\boldsymbol{\epsilon}_{\mathbf{q},q_z}\cdot\mathbf{x}|c\rangle$ is the dipole matrix element between valence and conduction band, $F_{\mathbf{q}}(0)$ is given by (2), and

$$I(q_z) = \int_{-\frac{L'}{2}}^{\frac{L'}{2}} \rho(z)\exp(iq_z z)dz. \quad (11)$$

To determine the expression of the $\mathbf{A}^2$ term in (4) we proceed as follows. The most general form for the second term on the right hand side of (4) is[14]

$$H_I^{(2)} = \frac{1}{2i\hbar}\frac{e^2}{c^2}\sum_i [\mathbf{A}(\mathbf{x}_i)\cdot\mathbf{x}_i, \mathbf{A}(\mathbf{x}_i)\cdot\mathbf{v}_i]. \quad (12)$$

To get expression (12) in second quantization form it is enough to take the expectation value over the exciton vacuum state. By introducing a sum over all the exciton states and using relation (6) we obtain

$$H_I^{(2)} = \frac{e^2}{\hbar c^2}\sum_{\mathbf{k}}\omega_{\mathbf{k}}|\langle 0|\sum_i \mathbf{A}(\mathbf{x}_i)\cdot\mathbf{x}_i|\Phi_{\mathbf{k}}\rangle|^2. \quad (13)$$

The quantity $\langle 0|\sum_i \mathbf{A}(\mathbf{x}_i)\cdot\mathbf{x}_i|\Phi_{\mathbf{k}}\rangle$ can be worked out as before. We finally get

$$H_I^{(2)} = \sum_{\mathbf{q}}\sum_{q_z,q_z'} D_{\mathbf{q},q_z,q_z'}(A^1_{\mathbf{q},q_z} + A^{1\dagger}_{-\mathbf{q},-q_z}) \times (A^1_{-\mathbf{q},-q_z'} + A^{1\dagger}_{\mathbf{q},q_z'}), \quad (14)$$

where $D_{\mathbf{q},q_z,q_z'}$ can be expressed in terms of the coefficients (10) as

$$D_{\mathbf{q},q_z,q_z'} = \frac{C_{\mathbf{q},q_z}C^*_{\mathbf{q},q_z'}}{\hbar\omega_{\mathbf{q}}}. \quad (15)$$

The whole Hamiltonian for the system given by radiation in interaction with the exciton field is thus the following:

$$H = \sum_{\mathbf{k}}\hbar\omega_{\mathbf{k}}A^{2\dagger}_{\mathbf{k}}A^2_{\mathbf{k}} + \sum_{\mathbf{q},q_z}\hbar v|\mathbf{q}+q_z\mathbf{z}|A^{1\dagger}_{\mathbf{q},q_z}A^1_{\mathbf{q},q_z} + H_I^{(1)} + H_I^{(2)}. \quad (16)$$

III. DISPERSION RELATIONS

In this section we derive the dispersion relations for upper and lower polaritons starting from the Hamiltonian (16). We define for this purpose the polariton operators

$$B_{\mathbf{q}} = \sum_{q_z} W(\mathbf{q},q_z)A^1_{\mathbf{q},q_z} + X(\mathbf{q})A^2_{\mathbf{q}} + \sum_{q_z} Y(\mathbf{q},q_z)A^{1\dagger}_{-\mathbf{q},-q_z} + Z(\mathbf{q})A^{2\dagger}_{-\mathbf{q}}. \quad (17)$$

The coefficients of the above expansion are chosen such that the Hamiltonian is diagonal in the $B_{\mathbf{q}}$ operators:

$$[B_{\mathbf{q}}, H] = EB_{\mathbf{q}}. \quad (18)$$

We substitute (17) and (16) in Eq. (18) and expand in terms of A^1, $A^{1\dagger}$, A^2, $A^{2\dagger}$ operators. By equating the coefficients corresponding to the same operator on the left and right hand sides, we get a system of equations which determine the coefficients in (17). The condition for this system to have solutions reduces to the following equation:

$$(\hbar\omega_{\mathbf{q}})^2 - E^2 = \frac{4E^2}{\hbar\omega_{\mathbf{q}}}\sum_{q_z}\frac{\hbar v\sqrt{q^2+q_z^2}|C_{\mathbf{q},q_z}|^2}{\hbar^2v^2(q^2+q_z^2)-E^2}, \quad (19)$$

which is the dispersion relation we were looking for. It is important to remark that, in contrast to the bulk case, in (19) a sum over q_z appears. The sum in (19) can be exploited analytically if we choose for the confinement function $\rho(z)$ a step function with values $1/L$ inside the QW and zero outside. This simplification is reasonable for a cavity width L' of the order of the wavelength corresponding to the exciton transition energy. In this case the electromagnetic modes interacting with the exciton will be with good approximation flat in the QW region, thus justifying the choice for $\rho(z)$. In addition we disregard the exciton dispersion, thus putting $\omega_{\mathbf{q}} = \omega_0$. Then, for the upper polariton modes, for which $E > \hbar vq$, we get

$$\omega^2 - 1 = \gamma\frac{\omega^2}{\alpha^3}\left\{k_0L\alpha + 2\cot\left(k_0\frac{L'}{2}\alpha\right)\sin^2\left(k_0\frac{L}{2}\alpha\right) - \sin(k_0L\alpha)\right\}. \quad (20)$$

In this expression we have introduced an effective interaction constant defined as

$$\gamma = 8\pi\frac{v}{c^2}\frac{|\mu|^2|F(0)|^2}{\hbar}\frac{1}{(k_0L)^2} \quad (21)$$

and the following normalized variables: $\omega = E/\hbar\omega_0$, $k_0 = \omega_0/v$, and $\alpha = \sqrt{|\omega^2 - q^2/k_0^2|}$. In (21) we disregard the $\mathbf{q}$ dependence of the exciton envelope function and of the dipole matrix element.

The analogous expression for the lower polariton mode, where $E < \hbar vq$, is given by

$$\omega^2 - 1 = -\gamma\frac{\omega^2}{\alpha^3}\left\{k_0 L\alpha + 2\coth\left(k_0\frac{L'}{2}\alpha\right)\sinh^2\left(k_0\frac{L}{2}\alpha\right) - \sinh(k_0 L\alpha)\right\}. \quad (22)$$

We plot the dispersion curves obtained from Eqs. (20) and (22) with a realistic choice of the parameters in Fig. 1. The curves on the left of the $\alpha = 0$ line are solutions of the upper polariton dispersion relation (20) while the curve on the right of the same line is the solution of the lower polariton dispersion relation (22). Where the exciton dispersion is resonant with one cavity mode the polariton dispersion curves show the typical anticrossing due to the interaction. This is visualized in the inset of Fig. 1.

With the model it is possible to fit the experimental data obtained by Weisbuch *et al.*[4] In that paper a $Al_{0.2}Ga_{0.8}As$ semiconductor microcavity was built, with five 76-Å GaAs QW's at its center. The geometry of the cavity is such that the thickness can be varied around the value corresponding to the exciton wavelength. The use of more than one QW allows to obtain a stronger interaction between excitons and cavity modes, because vacuum field Rabi splitting can be observed only in the strong interaction regime.[12] It can be shown that, under the assumption that each QW experiences the same electric field inside the cavity, the only change needed is a factor N multiplying (21), N being the number of QW's. Actually this assumption is not exactly verified in the experimental setup we are considering, since the array of QW's is not much smaller than the cavity thickness.

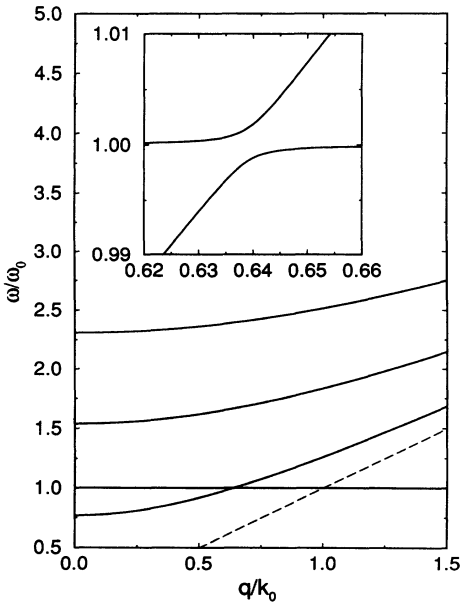

FIG. 1. Dispersion of polariton modes as obtained from Eqs. (20) and (22). The dashed line represents the $\alpha = 0$ line. Here $k_0 = \omega_0/v$. In the inset a detail containing the anticrossing point is shown. Material parameters appropriate to a $GaAs/Al_{0.2}Ga_{0.8}As$ QW and the cavity width $L' = 2.93 \times 10^{-5}$ cm are used.

This introduces an arbitrary factor in (21) that can be considered as an effective number of QW's smaller than N. Furthermore, the actual width of the cavity modes, not included in this model, will lower the interaction with respect to the one predicted by (21). For these reasons we prefer to use γ as a free parameter for making a fit of the experimental data. For this purpose we choose the following values for the other parameters: $\hbar\omega_0 = 1.586$ eV, and $\epsilon_0 = 12.0$. We calculate polariton energies for $\mathbf{q} = 0$ as a function of the detuning of the cavity, namely, of L'. The theoretical curves, compared to the experimental data in Ref. 4 are shown in Fig. 2. The value obtained for the constant, $\gamma = 8.2 \times 10^{-4}$, can be interpreted in terms of oscillator strength for the exciton and effective number of QW's. The following relation[1] is valid:

$$\frac{|\mu_{cv}|^2|F(0)|^2}{\hbar} = \frac{fe^2}{2m\omega_0}, \quad (23)$$

where f is the oscillator strength per unit surface of the exciton. The result obtained from the fit would correspond to an oscillator strength $f = 3 \times 10^{12}$ cm^{-2}, in very good agreement with theoretical predictions,[15] and to an effective QW's number of 1.04. This last result is reasonable in the light of the two effects mentioned above.

As a concluding remark of this section we would like to stress the role of the $\mathbf{A}^2$ term in the calculation. The results analogous to the ones obtained in this work, but without the $\mathbf{A}^2$ term, are already known.[13] In particular Eqs. (20) and (22) are still valid except that the ω^2 factor multiplying the right hand side is missing. The absence of this factor is determinant in the limit of large L'. If we look at the behavior of the lower branch, we can see that there is a finite value of L' for which this branch starts exactly from the origin of the dispersion

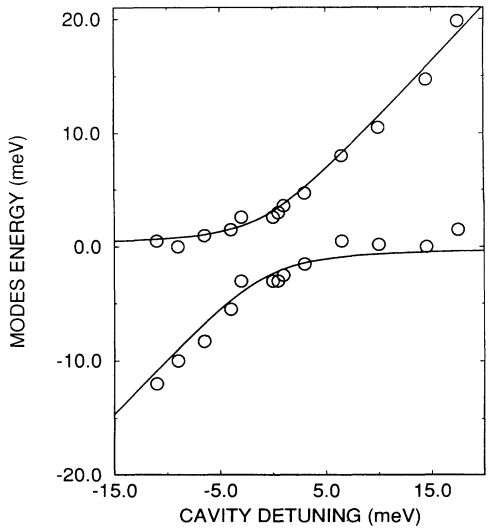

FIG. 2. Comparison between the experimental points from Ref. 4 and the theoretical results for the polariton frequency at $\mathbf{q} = 0$ as a function of the cavity detuning. The energy on the vertical axis is taken with respect to the exciton energy $\hbar\omega_0$. Material parameters are specified in the text.

plane. This would mean that a mode with zero energy exists in a confined system. Anyway, for such a system, we always expect a positive energy as a consequence of the confinement. With Eqs. (20) and (22), including the ω^2 factor, the correct behavior is recovered and the lower branch starting point approaches zero for $L' \to \infty$ as can be easily verified.

IV. LIMIT OF A LARGE CAVITY

In this section we investigate the limit of a large cavity, when the cavity modes become closer to each other. The results will allow a comparison with those obtained both classically[1] and quantum mechanically.[2,3,16]

Let us pay attention to the dispersion equation (20) in the case of large cavity. There are no problems with the lower polariton branch, because the only L' dependence in (22) appears in the function $\coth\left(k_0\frac{L'}{2}\alpha\right)$ which tends to one when $L' \to \infty$. Consequently, in this limit, the dispersion relation for the lower polariton branch reads

$$\omega^2 - 1 = -\gamma\frac{\omega^2}{\alpha^3}\left\{k_0 L\alpha - 1 + e^{-k_0 L\alpha)}\right\}. \tag{24}$$

This expression is identical to the classical result of Tassone *et al.*[1] calculated on the basis of the scattering theory. In this limit the starting point of the lower polariton curve in Fig. 3 moves towards the origin of the dispersion plane and the usual lower polariton curve is obtained.

To obtain the quantum counterpart of this result for upper polariton branches, a more careful treatment is necessary. There are several branches corresponding to different solutions of the dispersion relation (20), and

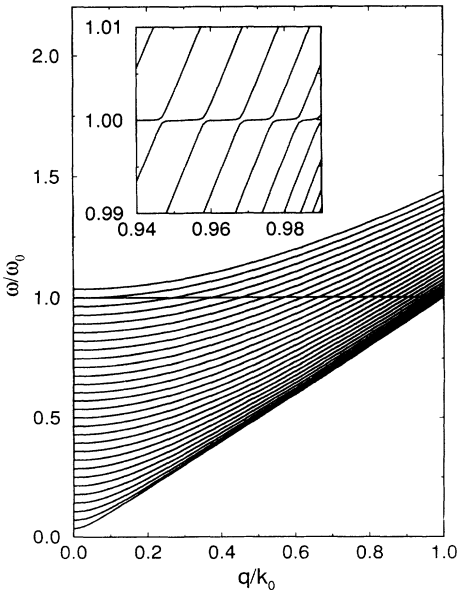

FIG. 3. The polariton dispersion curves in the case of a large cavity ($L' = 6.3 \times 10^{-4}$ cm). The inset shows a detail of the region close to the $\omega = \omega_0$ line. Material parameters as in Fig. 1.

when $L' \to \infty$, the set of branches in the upper half plane becomes dense. Nevertheless, the curves are not distributed uniformly and therefore a set of points with the highest concentration of upper polariton branches can be identified. This situation is depicted in Fig. 3. Let us discuss this point in terms of a density of states.

All the upper polariton curves may be advantageously parametrized using the variable α. The parametrization of the frequency ω is given using (20) as

$$\omega^2 = \left[1 - \frac{\gamma}{\alpha^3}\left\{k_0 L\alpha + 2\cot\left(k_0\frac{L'}{2}\alpha\right)\sin^2\left(k_0\frac{L}{2}\alpha\right) - \sin(k_0 L\alpha)\right\}\right]^{-1}. \tag{25}$$

The other variable q may be simply determined from the definition of α as

$$\frac{q^2}{k_0^2} = \left[1 - \frac{\gamma}{\alpha^3}\left\{k_0 L\alpha + 2\cot\left(k_0\frac{L'}{2}\alpha\right)\sin^2\left(k_0\frac{L}{2}\alpha\right) - \sin(k_0 L\alpha)\right\}\right]^{-1} - \alpha^2. \tag{26}$$

The range of definition of the parameter α is determined by the condition $q^2 \geq 0$. Let us specify the density of states. Assuming the isotropic model, the differential element corresponding to the two-dimensional vector $\mathbf{q}$ may be expressed as

$$2\pi q dq = \pi\frac{dq^2}{d\omega_j}d\omega_j,$$

where ω_j represents a given upper polariton branch. Consequently, the density of states may be specified along each branch as

$$\mathcal{D}_j = \frac{\pi}{k_0^2}\frac{dq^2}{d\omega_j}. \tag{27}$$

We look for the point on each branch, in which the density of states $\mathcal{D}_j$ is maximum. Such points can be visualized as inflection points on the different dispersion curves. Assuming L' finite but sufficiently large, the main contribution to the density of states may be explicitly specified as

$$\mathcal{D}_j = \frac{\pi}{k_0^2}\frac{dq_j^2}{d\omega_j} \approx 2\pi\omega_j\left[1 + \frac{4\alpha}{\Gamma\omega_j^2 k_0 L'[1+\cot^2\left(k_0\frac{L'}{2}\alpha\right)]}\right], \tag{28}$$

where the coefficient Γ is defined as

$$\Gamma = 16\pi\frac{v}{c^2}\frac{|\mu|^2|F(0)|^2}{\hbar}\frac{1}{\alpha^3}\frac{\omega^2}{(k_0 L)^2}\sin^2\left(k_0\frac{L}{2}\alpha\right).$$

In the derivation of (28) we used the fact that only the function $\cot\left(k_0\frac{L'}{2}\alpha\right)$ is changing significantly on each branch while all the other contributions may be neglected.

Using the dispersion relation (20) to express the term

$\cot^2\left(k_0\frac{L'}{2}\alpha\right)$, the density of states may be rewritten in the form

$$\mathcal{D}_{j,\text{approx}} = 2\pi\omega_j\left[1+\frac{4\alpha}{\omega_j^2 k_0 L'}\frac{\Gamma}{\Gamma^2+F^2}\right], \qquad (29)$$

where

$$F = \omega^2 - 1 - \gamma\frac{\omega^2}{\alpha^3}\{k_0 L\alpha - \sin(k_0 L\alpha)\}.$$

As long as L' is kept finite, the density of states is defined only on a finite set of curves. Points where the density of states has a maximum are located along the classical resonance curve $F \equiv 0$ independently of the scale of the cavity. The density of states is given by the Lorentzian profile of width corresponding to the classical resonance broadening. We conclude therefore that the lower polariton solution predicted by the the classical theory coincides with the one given in this paper for $L' \to \infty$. Moreover, the upper polariton branch predicted by the classical theory represents the common envelope of many dispersion curves existing inside the cavity.

V. CONCLUSIONS

In this paper we investigated the interaction of light with an exciton mode in a system formed by a QW placed inside a microcavity. A general form for the Hamiltonian has been derived using a microscopical model, including the $\mathbf{A}^2$ term. The main difference between the system here considered and the one of a free QW is that here, due to the confinement of light, the interaction with excitons in the QW is much stronger, the polariton effects thus being enhanced. A comparison with existing experimental data[4] has been made and the agreement is satisfactory. Moreover, we have shown in the last section how the well known results for a QW in an infinite medium are recovered by carrying out the limit of a very large cavity.

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[1] F. Tassone, F. Bassani, and L. C. Andreani, Nuovo Cimento D **12**, 1673 (1990).
[2] S. Jorda, U. Rössler, and D. Broido, Phys. Rev. B **48**, 1669 (1993).
[3] S. Jorda, U. Rössler, and D. Broido, Superlatt. Microstruct. **12**, 85 (1992).
[4] C. Weisbuch, M. Nishioka, A. Ishikawa, and Y. Arakawa, Phys. Rev. Lett. **69**, 3314 (1992).
[5] R. Houdre, R. P. Stanley, U. Oesterle, M. Ilegems, and C. Weisbuch, J. Phys. IV (France) Colloq. **3**, C5-51 (1993).
[6] J. J. Sanchez-Mondragon, N. B. Narozhny, and J. H. Eberly, Phys. Rev. Lett. **51**, 550 (1983).
[7] Y. Kaluzny, P. Goy, M. Gross, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **51**, 1175 (1983).
[8] Y. Zhu, D. J. Gauthier, S. E. Morin, Q. Wu, H. J. Carmichael, and T. W. Mossberg, Phys. Rev. Lett. **64**, 2499 (1990).
[9] R. J. Thompson, G. Rempe, and H. J. Kimble, Phys. Rev. Lett. **68**, 1132 (1992).
[10] A. Tredicucci, Y. Chen, and F. Bassani, Phys. Rev. B **47**, 10 348 (1993).
[11] Y. Chen, A. Tredicucci, and F. Bassani, J. Phys. (France) Colloq. **3**, C5-453 (1993).
[12] See, for instance, P. Meystre, in *Progress in Optics*, edited by E. Wolf (North-Holland, Amsterdam, 1992), Vol. 30, pp. 261–355.
[13] Z. Hradil, A. Quattropani, V. Savona, and P. Schwendimann, J. Phys. (France) Colloq. IV **3**, 393 C5-393 (1993).
[14] R. Girlanda, A. Quattropani, and P. Schwendimann, Phys. Rev. B **24**, 2009 (1981).
[15] L. C. Andreani and A. Pasquarello, Phys. Rev. B **42**, 8928 (1990).
[16] Z. Hradil, A. Quattropani, V. Savona, and P. Schwendimann, J. Stat. Phys. (to be published).