Kuželosečky

(1) ∞

$$2f = \ell + f - z \quad \Rightarrow \quad \ell = f + z$$

$$\ell^2 = r^2 + (f-z)^2$$

$$(f+z)^2 = r^2 + (f-z)^2$$

$$\cancel{f^2} + 2fz + \cancel{z^2} = r^2 + \cancel{f^2} - 2fz + \cancel{z^2}$$

$$\boxed{r^2 = 4fz = 2Rz}$$

ref. křivnice

parabola !

(2) s, s'

$$\ell + \ell' = s + s'$$

$$\ell^2 = r^2 + (s-z)^2$$

$$\ell'^2 = r^2 + (s'-z)^2$$

$$(\ell+\ell')(\ell-\ell') = \ell^2 - \ell'^2 = (s-z)^2 - (s'-z)^2$$

$$\left.\begin{aligned} \ell - \ell' &= \frac{(s-z)^2 - (s'-z)^2}{s+s'} \\ \ell + \ell' &= s + s' \end{aligned}\right\} \ell = \ell(s, s', z)$$

$$r^2 - 4z\frac{ss'}{s+s'} + 4z^2\frac{ss'}{(s+s')^2} = 0$$

$\boxed{\text{ODVOZENÍ}}$

$$\ell = \frac{(s-z)^2-(s'-z)^2}{2x} + \frac{x}{2}$$

$x = s+s'$

$y = s-s'$

$$= \frac{s^2-2sz+\cancel{z^2}-s'^2+2s'z-\cancel{z^2}}{2x} + \frac{x}{2}$$

$$= \frac{xy-2zy}{2x} + \frac{x}{2} = \frac{x+y}{2} - z\frac{y}{x}$$

$$r^2 = \ell^2 - (s-z)^2$$

$$= z^2\frac{y^2}{x^2} - z\frac{(x+y)y}{x} + \frac{(x+y)^2}{4} - s^2 + 2sz - z^2$$

$$r^2 = z^2\underbrace{\left(\frac{y^2}{x^2}-1\right)}_{1} + z\underbrace{\left(2s - \frac{(x+y)y}{x}\right)}_{2} + \underbrace{\frac{(x+y)^2}{4} - s^2}_{3}$$

3 $\quad \frac{(x+y)^2}{4} - s^2 = \frac{(s+s'+s-s')^2}{4} - s^2 = 0$

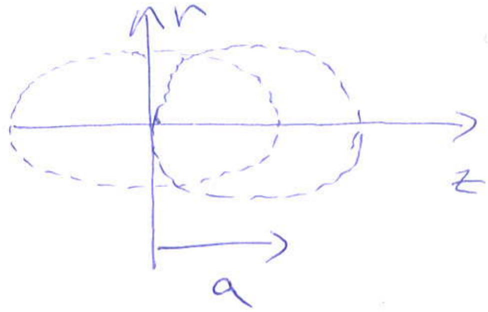
2 $\quad 2s - \frac{(x+y)y}{x} = \frac{2s(s+s')-2s(s-s')}{s+s'} = \frac{4ss'}{s+s'}$

1 $\quad \frac{y^2}{x^2} - 1 = \frac{y^2-x^2}{x^2} = \frac{s^2-2ss'+s'^2-s^2-2ss'-s'^2}{(s+s')^2}$

$$= -\frac{4ss'}{(s+s')^2}$$

$$\boxed{r^2 - z\frac{4ss'}{(s+s')} + z^2\frac{4ss'}{(s+s')^2} = 0}$$

rovnice elipsy $\frac{z^2}{a^2} + \frac{r^2}{b^2} = 1$



$$z \to z - a$$

$$\frac{z^2 - 2az + a^2}{a^2} + \frac{r^2}{b^2} = 1$$

$$\frac{z^2}{a^2} - 2\frac{z}{a} + 1 + \frac{r^2}{b^2} = 1$$

$$r^2 - 2z\frac{b^2}{a} + z^2\frac{b^2}{a^2} = 0$$

$$\Rightarrow \quad ss' = b^2$$
$$s + s' = 2a$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \quad \Rightarrow \quad \frac{ss'}{s+s'} = \frac{R}{2}$$

$$1 - e^2 = 1 - \frac{a^2 - b^2}{a^2} = 1 - 1 + \frac{b^2}{a^2} = \frac{4ss'}{(s+s')^2} \equiv 1 + K$$

$$\boxed{r^2 - 2Rz + (1+K)z^2 = 0}$$

$$K = -e^2$$

$e = 0 \Rightarrow K = 0$

$e > 1 \Rightarrow K < -1$

$0 < e < 1 \Rightarrow -1 < K < 0$

$e = 1 \Rightarrow K = -1$

umístění předmětu a obrazu

$$\left.\begin{aligned} s+s' &= 2a \\ ss' &= b^2 \end{aligned}\right\} \quad s' = \frac{b^2}{s}$$

$$s + \frac{b^2}{s} = 2a$$

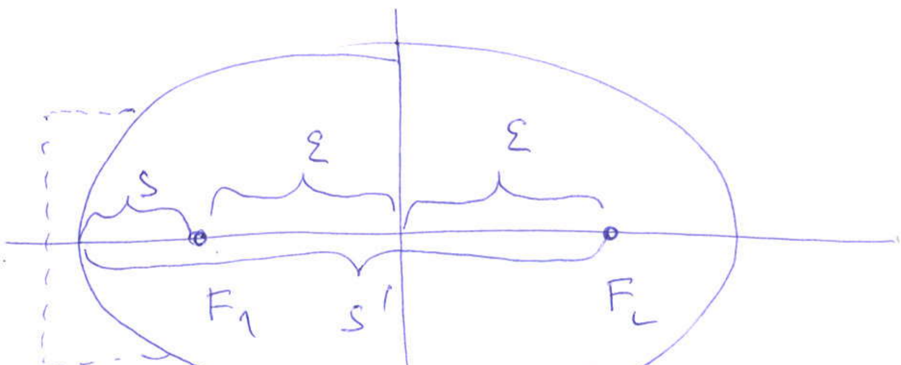
$$s^2 - 2as + b^2 = 0$$

$$s_{1,2} = \frac{2a \pm \sqrt{4a^2 - 4b^2}}{2}$$

$$s_{1,2} = a \pm \sqrt{a^2 - b^2}$$ (stejně pro $s'_{1,2}$)

$$s = a \pm \varepsilon$$

$$s' = a \mp \varepsilon$$



obraz, předmět v ohnisku elipsy

Kónická konstanta

$$1+K = \frac{4ss'}{(s+s')^2} \qquad m = -\frac{s'}{s} \Rightarrow s' = -ms$$

$$1+K = \frac{-4ms^2}{s^2(1-m)^2} = -\frac{4m}{(1-m)^2}$$

$$K = -\frac{4m}{(1-m)^2} - 1 = \frac{-4m-(1-m)^2}{(1-m)^2}$$

$$= \frac{-4m-1+2m-m^2}{(1-m)^2}$$

$$\boxed{K = -\left(\frac{m+1}{m-1}\right)^2}$$

profil zrcadla

$$r^2 - 2Rz + (1+K)z^2 = 0$$

$$z = \frac{2R \pm \sqrt{4R^2 - 4(1+K)r^2}}{2(1+K)}$$

$$z = \frac{R}{1+K}\left[1 - \sqrt{1 - \frac{r^2}{R^2}(1+K)}\right]$$

$$\sqrt{1-x} \cong 1 - \frac{x}{2} - \frac{x^2}{8}$$

$$z \cong \frac{R}{1+K}\left[1 - 1 + \frac{r^2}{2R^2}(1+K) + \frac{r^4}{8R^4}(1+K)^2\right]$$

$$z \cong \frac{r^2}{2R} + \frac{r^4}{8R^3}(1+K) + \dots$$

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max. odchylka sféry od paraboly

$$(\Delta z)_{max} = \frac{(D/2)^4}{8(2f)^3} = \frac{D^4}{16 \cdot 8 \cdot 8 f^3} = \frac{D}{1024\, c^3} = \frac{f}{1024\, c^4}$$

$D = 200\,mm$

$c = 8$

$$(\Delta z)_{max} = \frac{2 \cdot 10^{-1}}{10^3 \cdot 5 \cdot 10^2} = \underline{4 \cdot 10^{-7}\,m} \sim \lambda$$