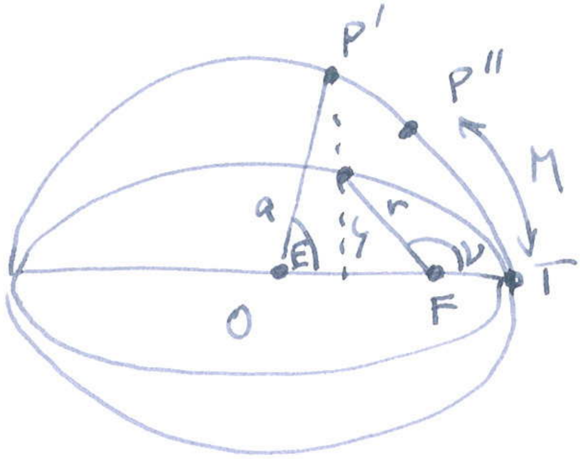
Keplerova rovnice



II Keplerův zákon

$$\frac{S_{TFP}}{\pi a b} = \frac{S_{TOP''}}{\pi a^2} = \frac{M}{2\pi} \Rightarrow S_{TFP} = \frac{ab}{2} M$$

projekce kružnice na elipsu

$$\frac{S_{TOP}}{\pi a b} = \frac{S_{TOP'}}{\pi a^2} = \frac{E}{2\pi}$$

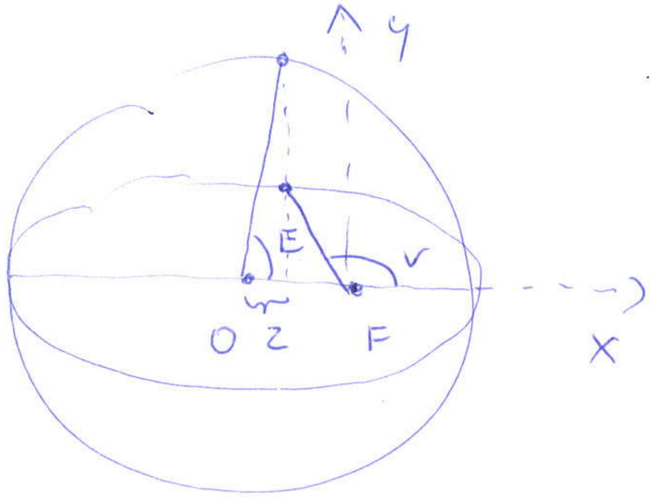
$$\Downarrow$$

$$E = \frac{2\left(\frac{ab}{2} M + \frac{\varepsilon y}{2}\right)}{ab} = M + \frac{\varepsilon y}{ab} = M + e\frac{y}{b}$$

$$\sin E = \frac{y\frac{a}{b}}{a} = \frac{y}{b}$$

$$\underline{\underline{E = M + e \sin E}}$$

poloha na dráze



$$a^2 = b^2 + a^2e^2$$

$$b^2 = a^2(1-e^2)$$

$$b = a\sqrt{1-e^2}$$

$$\frac{a}{b}\frac{y}{a} = \sin E \quad ; \quad y = b\sin E \quad \Rightarrow \quad \boxed{y = a\sqrt{1-e^2}\sin E}$$

$$\underbrace{\frac{z}{a} = \cos E \quad ; \quad x = z - \varepsilon}_{} = z - ae$$

$$x = a\cos E - ae \quad \Rightarrow \quad \boxed{x = a(\cos E - e)}$$

$$r = \sqrt{x^2+y^2} = \sqrt{a^2(\cos E - e)^2 + a^2(1-e^2)\sin^2 E}$$

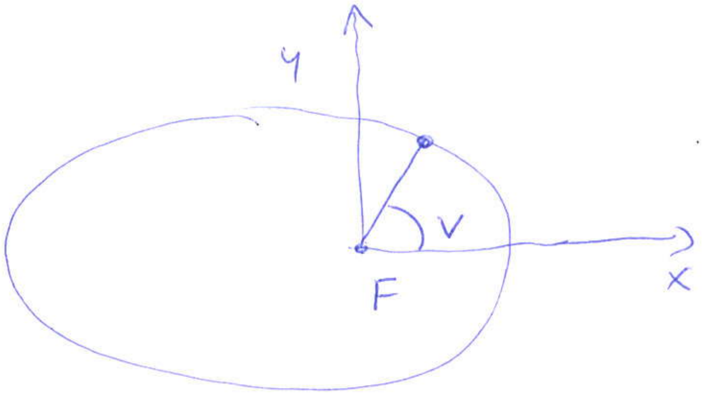
$$= a\sqrt{\cos^2 E - 2e\cos E + e^2 + \sin^2 E - e^2\sin^2 E}$$

$$= a\sqrt{1 + e^2(1-\sin^2 E) - 2e\cos E}$$

$$= a\sqrt{(1-e\cos E)^2}$$

$$\Rightarrow \quad \boxed{r = a(1 - e\cos E)}$$

pravá anomálie



$v \in [-\pi, \pi]$

$$\operatorname{tg}\frac{v}{2} = \frac{\sin v/2}{\cos v/2} = \frac{\pm\sqrt{\frac{1-\cos v}{2}}}{\sqrt{\frac{1+\cos v}{2}}} = \pm\sqrt{\frac{1-\cos v}{1+\cos v}}$$

$$\cos v = \frac{x}{r} = \frac{a(\cos E - e)}{a(1-e\cos E)} = \frac{\cos E - e}{1 - e\cos E}$$

$$\operatorname{tg}\frac{v}{2} = \pm\sqrt{\frac{\frac{1-e\cos E-\cos E+e}{1-e\cos E}}{\frac{1-e\cos E+\cos E-e}{1-e\cos E}}} = \pm\sqrt{\frac{(1+e)(1-\cos E)}{(1-e)(1+\cos E)}}$$

$$\operatorname{tg}\frac{v}{2} = \pm\sqrt{\frac{1+e}{1-e}}\sqrt{\frac{1-\cos E}{1+\cos E}}$$

$$\boxed{\operatorname{tg}\frac{v}{2} = \sqrt{\frac{1+e}{1-e}}\operatorname{tg}\frac{E}{2}}$$

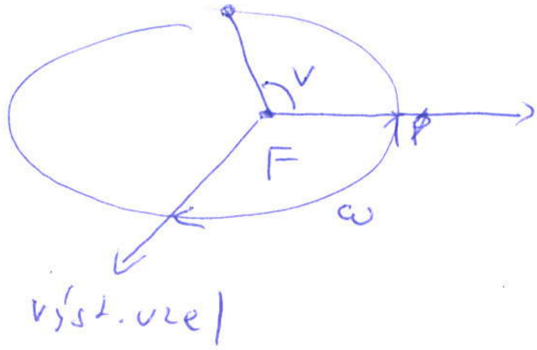
E a v vždy ve stejném kvadrantu tj. stejná znaménka

$v, E \in [-\pi, 0] \to (-)$

$v, E \in [0, \pi] \to (+)$

Pravá anomálie -> heliocentr. ekl.

1



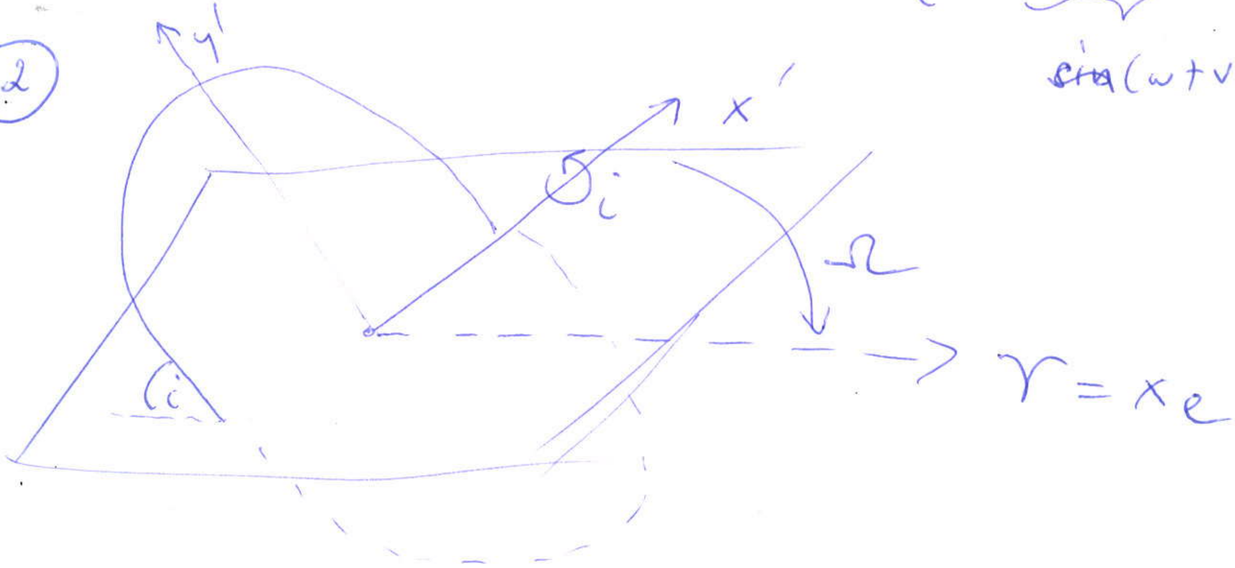
rotace v rovině pl. dráhy o ω

$$x = r\cos v \qquad y = r\sin v \quad \rightarrow \quad x' = r\cos(v+\omega) = r\cos\vartheta \qquad y' = r\sin(v+\omega) = r\sin\vartheta$$

$$\vartheta = v + \omega$$

tech. $$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\omega & -\sin\omega \\ +\sin\omega & \cos\omega \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} r(\overbrace{\cos\omega\cos v - \sin\omega\sin v}^{\cos(\omega+v)}) \\ r(\underbrace{\sin\omega\cos v + \cos\omega\sin v}_{\sin(\omega+v)}) \end{pmatrix}$$

2



~~rotace v rovině ekliptiky~~ o ~~Ω~~

rotace kolem osy x' o i

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{pmatrix} \begin{pmatrix} r\cos\theta \\ r\sin\theta \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = r \begin{pmatrix} \cos\theta \\ \cos i \sin\theta \\ \sin i \sin\theta \end{pmatrix}$$

③ rotace v rovině ekliptiky (kolem z'') o Ω

$$\begin{pmatrix} x_e \\ y_e \\ z_e \end{pmatrix} = \begin{pmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r\cos\theta \\ r\sin\theta\cos i \\ r\sin\theta\sin i \end{pmatrix}$$

$$\begin{pmatrix} x_e \\ y_e \\ z_e \end{pmatrix} = r \begin{pmatrix} \cos\Omega\cos\theta - \sin\Omega\sin\theta\cos i \\ \sin\Omega\cos\theta + \cos\Omega\sin\theta\cos i \\ \sin\theta\sin i \end{pmatrix}$$

$$\boxed{\begin{aligned} \frac{x_e}{r} &= \cancel{r}\cos\Omega\cos\theta - \sin\Omega\sin\theta\cos i \\ \frac{y_e}{r} &= \sin\Omega\cos\theta + \cos\Omega\sin\theta\cos i \\ \frac{z_e}{r} &= \sin\theta\sin i \end{aligned}}$$

pravoúhlé heliocentrické eklipt. souřadnice planety podobně vypočtu pravoúhlé helioc. ekl. souř. slunce

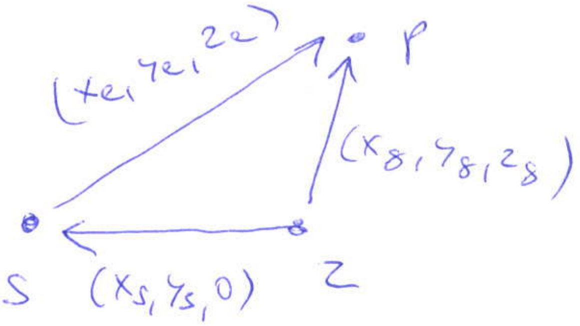
x_s, y_s, z_s

$$\boxed{\begin{aligned} x_s &= r_s \cos\theta_s \\ y_s &= r_s \sin\theta_s \\ z_s &= 0 \end{aligned}}$$

heliocentr. ekl. → geocentr. ekl.

$$\boxed{\begin{aligned} x_g &= x_e + x_s \\ y_g &= y_e + y_s \\ z_g &= z_e \end{aligned}} \rightarrow r_g$$

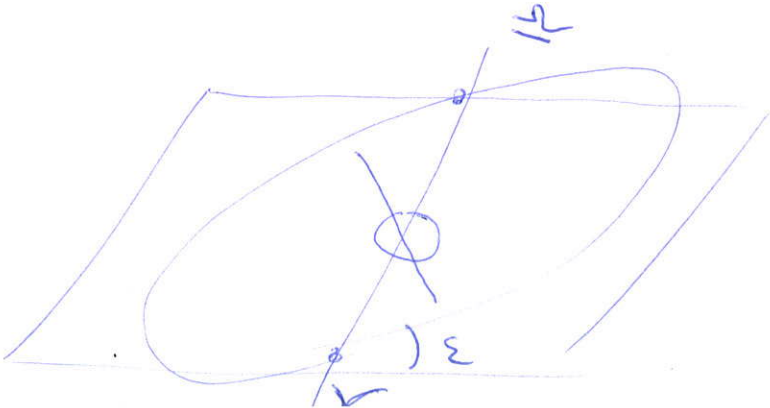
geoc. vzdál.



ekl. → rovníkové

= rotace kolem jarního bodu o ε (sklon ekl.)

$$\begin{pmatrix} x_q \\ y_q \\ z_q \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon & -\sin\varepsilon \\ 0 & \sin\varepsilon & \cos\varepsilon \end{pmatrix} \cdot \begin{pmatrix} x_g \\ y_g \\ z_g \end{pmatrix}$$



pravoúhlé → rovníkové II druhu

osa x míří do γ !

$$\Rightarrow \quad x_q = r \cos\alpha \cos\delta$$

$$y_q = r \sin\alpha \cos\delta$$

$$z_q = r \sin\delta$$

$$r = \sqrt{x_q^2 + y_q^2 + z_q^2}$$

$$\frac{x_q}{z_q} = \cos\alpha \frac{1}{\operatorname{tg}\delta} \quad ; \quad \frac{y_q}{z_q} = \sin\alpha \frac{1}{\operatorname{tg}\delta}$$

$$\boxed{\begin{aligned} \cos\alpha &= \operatorname{tg}\delta \frac{x_q}{z_q} \\ \sin\alpha &= \operatorname{tg}\delta \frac{y_q}{z_q} \\ \sin\delta &= \frac{z_q}{r} \end{aligned}}$$