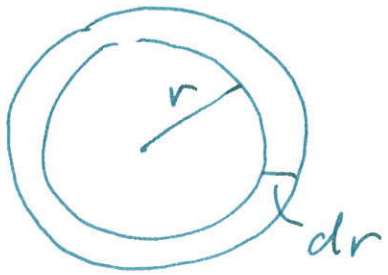
rovnice kontinuity



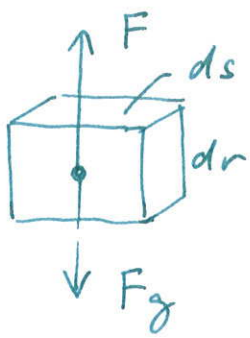
$$dm = 4\pi r^2 dr\, \rho$$

$$\boxed{\frac{dm}{dr} = 4\pi r^2 \rho}$$

$$\frac{M}{R} \sim R^2 \rho$$

$$\boxed{\rho \sim M R^{-3}}$$

hydrostatická rovnováha



$$F_g = \frac{G\,ds\,dr\,\rho\,m}{r^2}$$

$$P(r+dr) = P(r) + dP$$

$$F = [P(r) - P(r+dr)]\,ds = -dP\,ds$$

$$F_g = F$$

$$\Downarrow$$

$$dP = -\frac{G\,dr\,\rho\,m}{r^2}$$

$$\boxed{\frac{dP}{dr} = -\frac{G\rho m}{r^2}}$$

$$\frac{P_c}{R} \sim \frac{GMR^{-3}M}{R^2}$$

$$\boxed{P \sim M^2 R^{-4}}$$

Slunce:

$$P_c \sim \frac{GM^2}{R^4} = \frac{6\cdot 10^{-11}\cdot 4\cdot 10^{60}}{(10^9)^4} = \frac{24\cdot 10^{49}}{10^{36}} = 2\cdot 10^{50}\cdot 10^{-36}$$

$$P_c \sim 2\cdot 10^{14}\,P_a$$

Stavová rovnice

$$P = n_p k T$$

(n_p ... koncentrace)

$$n_p = \frac{N_p}{V} = \frac{m}{m_p}\frac{1}{V} = \frac{\rho}{m_p}$$

$$P = \frac{\rho}{m_p} k T$$

$$\Downarrow$$

$$T = \frac{m_p P}{\rho k} \sim \frac{M^2}{R^3} M^{-1} R^3$$

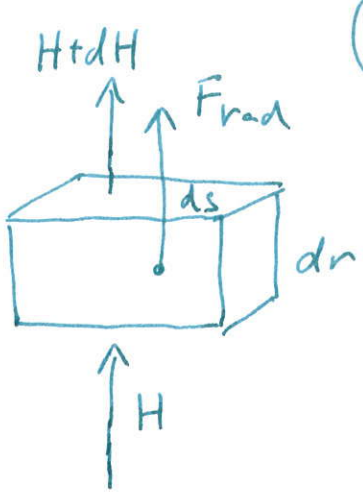
$$\boxed{T \sim \frac{M}{R}}$$

$$\left.\begin{aligned} T &\sim \frac{M}{R} \\ \rho &\sim \frac{M}{R^3} \end{aligned}\right\} \quad \rho \sim \frac{T^3}{M^2} \qquad (\rho\text{-}T \text{ diagram})$$

Slunce:

$$T_c = \frac{10^{-27}\, 10^{14}}{10^{3}\, 10^{-23}} = \frac{10^{-13}}{10^{-20}} \sim 10^7 K$$

Zářivý transport



$$dH = -\kappa H \rho \, dr \quad [W m^{-2}]$$

$$E_{fot} = \hbar\omega, \quad p_{fot} = \hbar k, \quad \frac{\omega}{k} = c$$

$$\Rightarrow \quad p = \frac{E}{c} \quad \text{– energie}$$

hybnost

(záporný přírustek hybnosti záření)

$$dP_{rad} = -\frac{F_{rad}}{ds} = \frac{dp}{ds\,dt} = \frac{dE}{c\,ds\,dt} = \frac{dH}{c}$$

$$P_{rad} = \frac{1}{3} a T^4, \quad a = \frac{4\sigma}{c} \quad \text{– S-B konst.}$$

$$\frac{dP_{rad}}{dr} = \frac{1}{3} a 4 T^3 \frac{dT}{dr}$$

$$H = -\frac{dH}{\kappa \rho \, dr} = -\frac{c}{\kappa\rho}\frac{dP_{rad}}{dr} = -\frac{4acT^3}{3\kappa\rho}\frac{dT}{dr}$$

$$\boxed{F = -4\pi r^2 \frac{4acT^3}{3\kappa\rho}\frac{dT}{dr}}$$ tok hvězdou

$$F \sim \frac{R^2 T^3}{\rho}\frac{T}{R} = R^2 \frac{M^4}{R^4} M^{-1} R^3 R^{-1} = M^3$$

$$\boxed{L \sim M^3}$$ tok povrchem

$$F(r+dr) = F(r) + 4\pi r^2 dr\, \rho\, q$$ (R)

$$F(r+dr) = F(r) + \frac{dF}{dr} \cdot dr$$

$$\Rightarrow \boxed{\frac{dF}{dr} = 4\pi r^2 \rho\, q}$$ tepelná rovnováha

$$q \sim \rho T^n$$

$$M^3 R^{-1} = R^2 M R^{-3} M R^{-3} \underbrace{T^n}_{M^n R^{-n}} = M^2 R^{-4} M^n R^{-n}$$

$$M^{3-2-n} = R^{-4+1-n}$$

$$R^{-3-n} = M^{1-n}$$

$$R = M^{\frac{n-1}{n+3}}$$

CNO $n = 16$ $\Rightarrow$ $R \sim M$

P-P $n = 4$ $\Rightarrow$ $R \sim M^{3/7}$

Černé těleso

T_{ess}

$$L = H \cdot S \sim T_{ess}^4 R^2$$

$$T_{ess} \sim L^{1/4} R^{-1/2}$$

CNO

$$L \sim M^3, \quad R \sim M$$

$$T_{ess} \sim M^{3/4} M^{-1/2} = \underline{M^{1/4}}$$

P-P

$$L \sim M^3, \quad R \sim M^{3/7} \sim M^{1/2}$$

$$T_{ess} \sim M^{3/4} M^{-\frac{1}{2}\frac{1}{2}} = M^{\frac{3}{5}} M^{-\frac{1}{4}}$$

$$\underline{T_{ess} \sim M^{\frac{1}{2}}}$$

H-R

$$L R^2 \sim T_{eff}^4$$

$$\left.\begin{aligned} R &\sim M^{\frac{n-1}{n+3}} \\ L &\sim M^3 \end{aligned}\right\} \quad R \sim L^{\frac{n-1}{3(n+3)}}$$

$$L \cdot L^{-\frac{2(n-1)}{3(n+3)}} \sim T_{eff}^4$$

$$L^{1-\frac{2(n-1)}{3(n+3)}} \sim T_{eff}^4$$

$$\left[1-\frac{2(n-1)}{3(n+3)}\right] \lg L \sim 4 \lg T_{eff}$$

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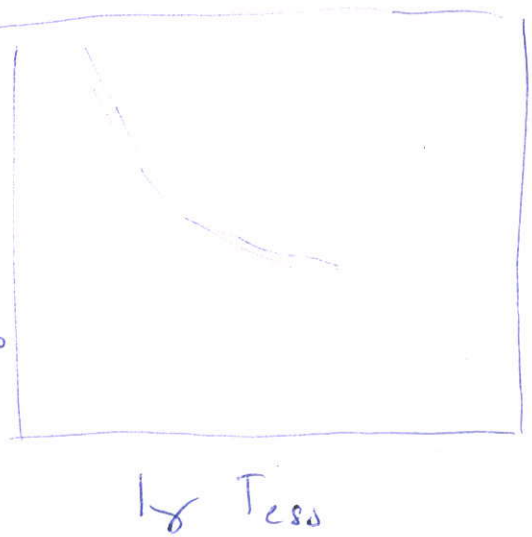
① P-P $n = 4$

$\lg L \sim 5{,}6 \lg T_{eff}$

$\lg L \sim \frac{12(n+3)}{n+11} \lg T_{eff}$

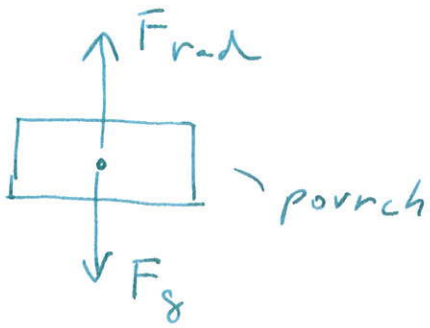
② CNO $n = 16$

$\lg L \sim 8{,}4 \lg T_{eff}$



Eddingtonova luminosita

rad. tlak = gravitace



$$F_g = \frac{GM\,dr\,ds\,\rho}{R^2}$$

$$F_{rad} = -\frac{dH}{c}\,ds = \frac{\kappa H \rho\,dr\,ds}{c}$$

$$F_g = F_{rad}$$

$$\Downarrow$$

$$\frac{\kappa H}{c} = \frac{GM}{R^2}$$

$$H = \frac{GMc}{\kappa R^2}$$

$$\boxed{L = 4\pi R^2 \frac{GMc}{\kappa R^2} = \frac{4\pi GMc}{\kappa}}$$

Eddington luminosity

$$L = \frac{M^3}{M_\odot^3} L_\odot < \frac{4\pi c G M}{\kappa}$$

$$M_\odot = 2 \cdot 10^{30}$$

$$L_\odot = 3{,}85 \cdot 10^{26}$$

$$\frac{M^2}{M_\odot^2} < \frac{4\pi c G}{\kappa L_\odot} M_\odot$$

$$\kappa_{es} = 4 \cdot 10^{-2} \quad (X = 1)$$

$$\left(\frac{M}{M_\odot}\right)^2 < \frac{4\pi c G}{\kappa} \frac{M_\odot}{L_\odot}$$

$$\kappa_{es} = 3{,}4 \cdot 10^{-2} \quad (X = 0{,}7)$$

$$\left(\frac{M}{M_\odot}\right)_{max} = 180$$

$$X = 0{,}7 \Rightarrow \left(\frac{M}{M_\odot}\right)_{max} = 200$$

p-p řetězec

$$p + p \rightarrow {}^{2}D + e^{+} + \nu \qquad 2\times$$

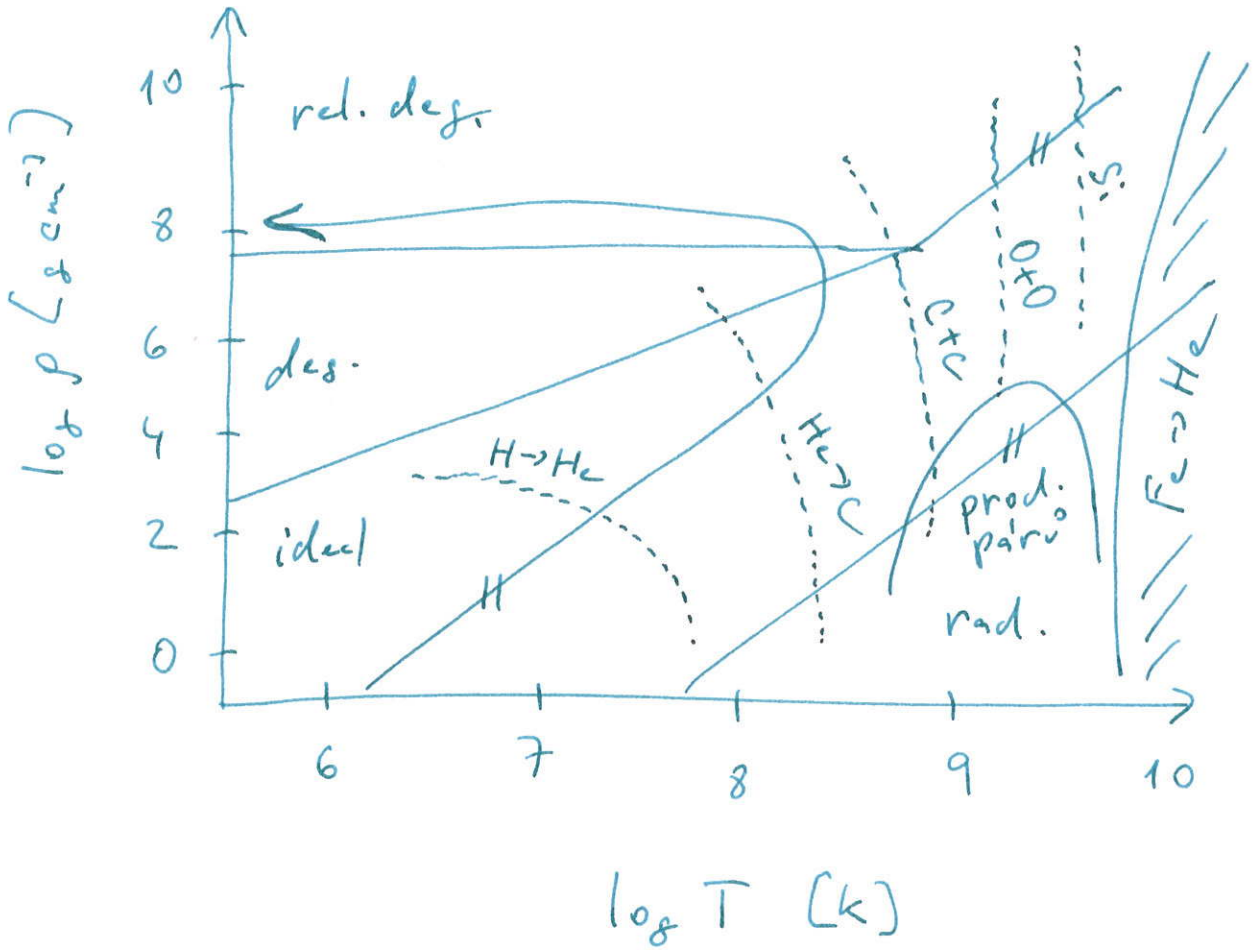
$${}^{2}D + p \rightarrow {}^{3}He + \gamma \qquad 2\times$$

$${}^{3}He + {}^{3}He \rightarrow {}^{4}He + 2p$$

$$4\,p \rightarrow {}^{4}He + 2e^{+} + 2\nu + 2\gamma$$

- $2e^{+}$ — anihilace
- 2ν — ven
- 2γ — záření

ρ - T diagram


10
8
6
4
2
0
rel. deg.
deg.
ideal
H→He
He→C
C+C
O+O
Si
prod. pair
rad.
Fe→He
6
7
8
9
10
log ρ [g cm⁻³]
log T [K]