Numerické metody a programování

Lekce 1

Numerické metody a programování

Obsah přednášky

1. Mathematica: základy programování, symbolické výpočty, vizualizace dat.
2. Programování v prostředích Matlab/Octave.
3. Úvod do numerických metod: přesnost, zaokrouhlovací chyby, stabilita.
4. Lineární algebra.
5. Interpolace a extrapolace.
6. Integrace funkcí a řešení obyčejných diferenciálních rovnic.
7. Řešení soustav nelineárních rovnic.
8. Optimalizace.
9. Diskrétní Fourierova transformace, FFT.
10. Aplikace FFT v optice.
11. Modelování dat.

Doporučená literatura

- E. Vitásek, Numerické metody (SNTL, Praha, 1987).
- W.H. Press, S.A. Teukolsky, W.T. Vetterling, B.P. Flannery, Numerical Recipes in C, (Cambridge University Press, Cambridge, 1992); dostupné online na http://www.nr.com
- J.D. Schmidt, Numerical Simulation of Optical Wave Propagation (SPIE Press, 2010)
- Manuály Matlab/Octave (http://www.octave.cz), Mathematica, Oslo (http://www.lambdares.com)

Mathematica (Wolfram Research)
http://www.wolfram.com/mathematica/

- symbolické výpočty
- numerické výpočty
- vizualizace dat a výsledků

```
(* prirazeni, relace *)

In[1]:= a = 1
Out[1]= 1

In[2]:= a
Out[2]= 1

In[3]:= a =.

In[4]:= a
Out[4]= a

In[5]:= a = 1
Out[5]= 1

In[6]:= a = a + 1
Out[6]= 2

In[7]:= a++;

In[8]:= a
Out[8]= 3

In[9]:= a += 5
Out[9]= 8

In[10]:= b = 1
Out[10]= 1

In[11]:= a > b
Out[11]= True

In[12]:= a == b
Out[12]= False


(* komplexni cisla *)

In[13]:= x = 2 + I
Out[13]= 2 + i

In[14]:= x^2
Out[14]= 3 + 4 i

In[15]:= Re[x]
Out[15]= 2

In[16]:= Im[x]
Out[16]= 1
```

```
In[17]:= Abs[x]
Out[17]= √5

In[18]:= Arg[x]
Out[18]= ArcTan[1/2]

In[19]:= Conjugate[x]
Out[19]= 2 - i

In[20]:= x =.

In[21]:= f = x^2 - Abs[x]^2
Out[21]= x^2 - Abs[x]^2

In[22]:= Simplify[f]
Out[22]= x^2 - Abs[x]^2

In[23]:= Simplify[f, Im[x] == 0]
Out[23]= 0


         (* ridici struktury *)

In[24]:= a = 2; b = 4;

In[25]:= If[a < b, mensi = a, mensi = b];

In[26]:= mensi
Out[26]= 2

In[27]:= For[i = 1, i ≤ 10, i ++, Print[i]]
         1
         2
         3
         4
         5
         6
         7
         8
         9
         10

In[28]:= suma = 0
Out[28]= 0

In[29]:= For[i = 1, i ≤ 10, i ++, suma += i]
         Print[suma]
         55
```

In[31]:=
```
suma = 0; i = 1;
While[i ≤ 10,
 suma += i;
 i += 2]
Print[suma]
```

25

In[34]:=
```
min[a_, b_] := If[a < b, a, b]
```

In[35]:=
```
min[1, 2]
```

Out[35]= 1

In[36]:=
```
min[3, 4]
```

Out[36]= 3

```
(* vektory, matice *)
```

In[37]:=
```
v = {1, 2, 3}
```

Out[37]= {1, 2, 3}

In[38]:=
```
MatrixForm[v]
```

Out[38]//MatrixForm=

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

In[39]:=
```
Sqrt[v]
```

Out[39]= $\{1, \sqrt{2}, \sqrt{3}\}$

In[40]:=
```
v.v
```

Out[40]= 14

In[41]:=
```
v = Table[Cos[x], {x, 0, 2 π, π / 2}]
```

Out[41]= {1, 0, -1, 0, 1}

In[42]:=
```
v2 = Table[x, {x, 0, 2 π, π / 2}]
```

Out[42]= $\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\}$

In[43]:=
```
Cos[v2]
```

Out[43]= {1, 0, -1, 0, 1}

In[44]:=
```
m = RandomReal[{0, 1}, {3, 3}]
```

Out[44]= {{0.711135, 0.246579, 0.197264},
 {0.216202, 0.870612, 0.0599256}, {0.522712, 0.379262, 0.0271403}}

In[45]:=
```
MatrixForm[m]
```

Out[45]//MatrixForm=

$$\begin{pmatrix} 0.711135 & 0.246579 & 0.197264 \\ 0.216202 & 0.870612 & 0.0599256 \\ 0.522712 & 0.379262 & 0.0271403 \end{pmatrix}$$

In[46]:=
```
m[[All, 1]]
```
Out[46]=
```
{0.711135, 0.216202, 0.522712}
```

In[47]:=
```
m[[2, All]]
```
Out[47]=
```
{0.216202, 0.870612, 0.0599256}
```

In[48]:=
```
m[[1 ;; 2, 1 ;; 2]] // MatrixForm
```
Out[48]//MatrixForm=

$$\begin{pmatrix} 0.711135 & 0.246579 \\ 0.216202 & 0.870612 \end{pmatrix}$$

In[49]:=
```
ConstantArray[0, {5, 5}] // MatrixForm
```
Out[49]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[50]:=
```
Table[0, {i, 1, 5}, {j, 1, 5}] // MatrixForm
```
Out[50]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[51]:=
```
m = {{2, 1}, {-1, 3}}
```
Out[51]=
```
{{2, 1}, {-1, 3}}
```

In[52]:=
```
MatrixForm[m]
```
Out[52]//MatrixForm=

$$\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$$

In[53]:=
```
m m
```
Out[53]=
```
{{4, 1}, {1, 9}}
```

In[54]:=
```
{{4, 1}, {1, 9}} // MatrixForm
```
Out[54]//MatrixForm=

$$\begin{pmatrix} 4 & 1 \\ 1 & 9 \end{pmatrix}$$

In[55]:=
```
m.m // MatrixForm
```
Out[55]//MatrixForm=

$$\begin{pmatrix} 3 & 5 \\ -5 & 8 \end{pmatrix}$$

In[56]:=
```
v = {{1}, {1}}
```
Out[56]=
```
{{1}, {1}}
```

In[57]:=
```
MatrixForm[v]
```
Out[57]//MatrixForm=

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

```
In[58]:= MatrixForm[m.v]
```

Out[58]//MatrixForm=

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

```
In[59]:= a = {{a11, a12}, {a21, a22}}
Out[59]= {{a11, a12}, {a21, a22}}

In[60]:= b = {{b11, b12}, {b21, b22}}
Out[60]= {{b11, b12}, {b21, b22}}

In[61]:= c = a.b; c // MatrixForm
```

Out[61]//MatrixForm=

$$\begin{pmatrix} a11\, b11 + a12\, b21 & a11\, b12 + a12\, b22 \\ a21\, b11 + a22\, b21 & a21\, b12 + a22\, b22 \end{pmatrix}$$

```
In[62]:= Det[a]
Out[62]= -a12 a21 + a11 a22

In[63]:= Tr[a]
Out[63]= a11 + a22

(* linearni algebra *)

In[64]:= a = {{0, 1, 1}, {1, 1, 1}, {1, 1, 0}}; a // MatrixForm
```

Out[64]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

```
In[65]:= eig = Eigenvalues[a]
```

Out[65]= $\left\{1 + \sqrt{2}, -1, 1 - \sqrt{2}\right\}$

```
In[66]:= vec = Eigenvectors[a]
```

Out[66]= $\left\{\left\{1, -\frac{-2 - \sqrt{2}}{1 + \sqrt{2}}, 1\right\}, \{-1, 0, 1\}, \left\{1, -\frac{2 - \sqrt{2}}{-1 + \sqrt{2}}, 1\right\}\right\}$

```
In[67]:= a.vec[[1]]
```

Out[67]= $\left\{1 - \frac{-2 - \sqrt{2}}{1 + \sqrt{2}}, 2 - \frac{-2 - \sqrt{2}}{1 + \sqrt{2}}, 1 - \frac{-2 - \sqrt{2}}{1 + \sqrt{2}}\right\}$

```
In[68]:= eig[[1]] vec[[1]]
```

Out[68]= $\left\{1 + \sqrt{2}, 2 + \sqrt{2}, 1 + \sqrt{2}\right\}$

```
In[69]:= b = Inverse[a]; b // MatrixForm
```

Out[69]//MatrixForm=

$$\begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

In[70]:=
```
a.b // MatrixForm
```
Out[70]//MatrixForm=
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[71]:=
```
prava = {1, 2, 3}
```
Out[71]= {1, 2, 3}

In[72]:=
```
x = b.prava
```
Out[72]= {1, 2, -1}

In[73]:=
```
a.x == prava
```
Out[73]= True

```
(* vyrazy *)
```

In[74]:=
```
a =.; b =.; c =.; x =.;
```

In[75]:=
```
f1 = x
```
Out[75]= x

In[76]:=
```
f2 = Exp[-x]
```
Out[76]= e^{-x}

In[77]:=
```
f = f1 * f2
```
Out[77]= $e^{-x}\,x$

```
(* derivace *)
```

In[78]:=
```
D[f, x]
```
Out[78]= $e^{-x} - e^{-x}\,x$

In[79]:=
```
D[f, x, x]
```
Out[79]= $-2\,e^{-x} + e^{-x}\,x$

```
(* integrace *)
```

In[80]:=
```
integral = Integrate[f, x]
```
Out[80]= $e^{-x}\,(-1 - x)$

In[81]:=
```
tem = D[integral, x]
```
Out[81]= $-e^{-x} - e^{-x}\,(-1 - x)$

In[82]:=
```
simp = Simplify[tem]
```
Out[82]= $e^{-x}\,x$

In[83]:= `simp == f`

Out[83]= `True`

In[84]:= `Integrate[Exp[-x^2], {x, -Infinity, Infinity}]`

Out[84]= $\sqrt{\pi}$

In[85]:= `vysl = Integrate[Exp[-a x^2], {x, -Infinity, Infinity}]`

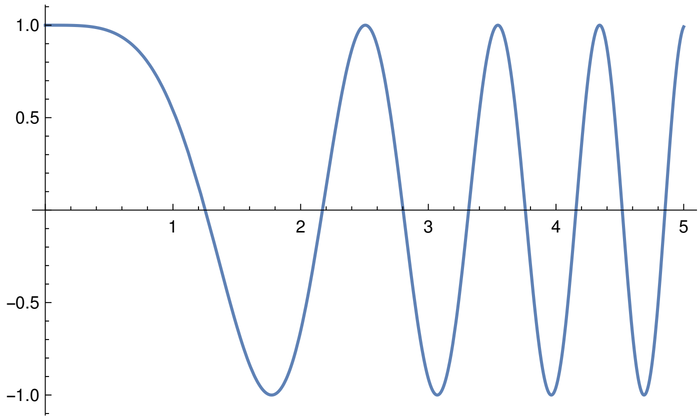
Out[85]= ConditionalExpression$\left[\frac{\sqrt{\pi}}{\sqrt{a}}, \text{Re}[a] > 0\right]$

In[86]:= `Simplify[vysl, a > 0]`

Out[86]= $\frac{\sqrt{\pi}}{\sqrt{a}}$

In[87]:= `Plot[Cos[x^2], {x, 0, 5}]`

Out[87]=

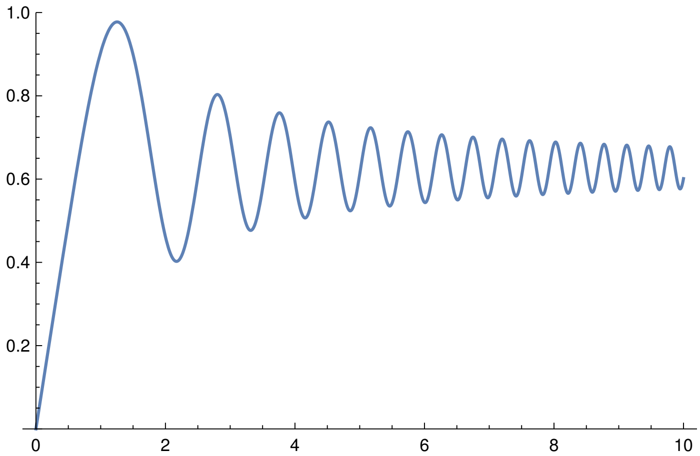


In[88]:= `res = Integrate[Cos[x^2], {x, 0, α}]`

Out[88]= $\sqrt{\frac{\pi}{2}}$ FresnelC$\left[\sqrt{\frac{2}{\pi}}\ \alpha\right]$

In[89]:= `Plot[res, {α, 0, 10}, PlotRange → {0, 1}, PlotPoints → 50]`

Out[89]=



In[90]:= `Integrate[Cos[x^2], {x, 0, 1}] // N`

Out[90]= `0.904524`

In[91]:= `NIntegrate[Cos[x^2], {x, 0, 1}]`

Out[91]= 0.904524

In[92]:= `i1 = Integrate[Cos[x^2], {x, 0, 100}] // N`

Out[92]= 0.625129

In[93]:= `NIntegrate[Cos[x^2], {x, 0, 100}]`

NIntegrate::ncvb :
NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in x near {x} = {0.}. NIntegrate obtained 0.4808197282114536` and 0.0116887984970139` for the integral and error estimates. ≫

Out[93]= 0.48082

In[94]:= `i2 = NIntegrate[Cos[x^2], {x, 0, 100}, MaxRecursion → 10]`

Out[94]= 0.625129

In[95]:= `i1 - i2`

Out[95]= 8.10463×10^{-15}

`(* soucty rad *)`

In[96]:= `Sum[n^2, {n, 1, 3}]`

Out[96]= 14

In[97]:= `Sum[1 / n^2, {n, 1, Infinity}]`

Out[97]= $\frac{\pi^2}{6}$

In[98]:= `Sum[1 / 2^n, {n, 0, Infinity}]`

Out[98]= 2

In[99]:= `Simplify[Sum[n, {n, 1, a}]]`

Out[99]= $\frac{1}{2} a (1 + a)$

`(* rovnice *)`

In[100]:= `Solve[2 x + 5 == 0, x]`

Out[100]= $\left\{\left\{x \to -\frac{5}{2}\right\}\right\}$

In[101]:= `Solve[{2 x + y == 1, x - y == 2}, {x, y}]`

Out[101]= $\{\{x \to 1, y \to -1\}\}$

In[102]:= `Solve[a x^2 + b x + c == 0, x]`

Out[102]= $\left\{\left\{x \to \frac{-b - \sqrt{b^2 - 4 a c}}{2 a}\right\}, \left\{x \to \frac{-b + \sqrt{b^2 - 4 a c}}{2 a}\right\}\right\}$

```
In[103]:= f = Expand[(x - 2) (x - 1) (x + 2)]
Out[103]= 4 - 4 x - x^2 + x^3

In[104]:= Solve[f == 0, x]
Out[104]= {{x → -2}, {x → 1}, {x → 2}}

In[105]:= f = Cos[x] - x
Out[105]= -x + Cos[x]

In[106]:= Solve[f == 0, x]
          Solve::nsmet : This system cannot be solved with the methods available to Solve. ≫
Out[106]= Solve[-x + Cos[x] == 0, x]

In[107]:= vysl = FindRoot[f, {x, -1}]
Out[107]= {x → 0.739085}

In[108]:= Cos[vysl[[1, 2]]]
Out[108]= 0.739085


          (* diferencialni rovnice *)

In[109]:= DSolve[y'[x] + y[x] == a Sin[x], y[x], x]
```

Out[109]= $\left\{\left\{y[x] \to e^{-x} C[1] + \frac{1}{2} a \left(-Cos[x] + Sin[x]\right)\right\}\right\}$

```
In[110]:= DSolve[y''[x] + k^2 y[x] == 0, y[x], x]
Out[110]= {{y[x] → C[1] Cos[k x] + C[2] Sin[k x]}}

In[111]:= DSolve[{y''[x] + k^2 y[x] == 0, y[0] == 1}, y[x], x]
Out[111]= {{y[x] → Cos[k x] + C[2] Sin[k x]}}

In[112]:= DSolve[{y''[x] + k^2 y[x] == 0, y[0] == 1, y'[0] == 0}, y[x], x]
Out[112]= {{y[x] → Cos[k x]}}


          (* trigonometricke funkce *)

In[113]:= Cos[x + y]
Out[113]= Cos[x + y]

In[114]:= vysl = TrigExpand[Cos[x + y]]
Out[114]= Cos[x] Cos[y] - Sin[x] Sin[y]

In[115]:= TrigFactor[vysl]
Out[115]= Cos[x + y]
```

In[116]:= `f = (n1 Cos[α2] - n2 Cos[α1]) / (n1 Cos[α2] + n2 Cos[α1])`

Out[116]= $\frac{-n2\,\mathrm{Cos}[\alpha 1] + n1\,\mathrm{Cos}[\alpha 2]}{n2\,\mathrm{Cos}[\alpha 1] + n1\,\mathrm{Cos}[\alpha 2]}$

In[117]:= `n1 = n2 Sin[α2] / Sin[α1]`

Out[117]= n2 Csc[α1] Sin[α2]

In[118]:= `f2 = Simplify[f]`

Out[118]= $\frac{-\mathrm{Sin}[2\,\alpha 1] + \mathrm{Sin}[2\,\alpha 2]}{\mathrm{Sin}[2\,\alpha 1] + \mathrm{Sin}[2\,\alpha 2]}$

In[119]:= `TrigFactor[f2]`

Out[119]= -Cot[α1 + α2] Tan[α1 - α2]

In[120]:= `TrigToExp[Sin[x]]`

Out[120]= $\frac{1}{2}\,i\,e^{-i\,x} - \frac{1}{2}\,i\,e^{i\,x}$

In[121]:= `ExpToTrig[Exp[I x]]`

Out[121]= Cos[x] + i Sin[x]

`(* rozvoj v radu *)`

In[122]:= `Series[Exp[x], {x, 0, 3}]`

Out[122]= $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + O[x]^4$

In[123]:= `Series[Sqrt[1 + x], {x, 0, 3}]`

Out[123]= $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + O[x]^4$

In[124]:= `Series[(n + 2) / (n + 3), {n, Infinity, 3}]`

Out[124]= $1 - \frac{1}{n} + \frac{3}{n^2} - \frac{9}{n^3} + O\left[\frac{1}{n}\right]^4$

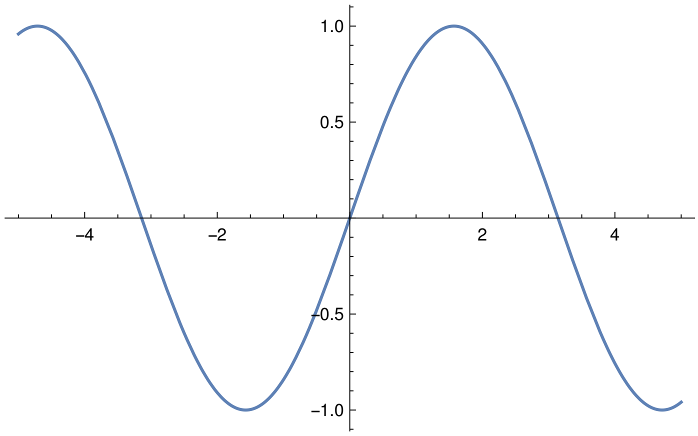
In[125]:= `Normal[%]`

Out[125]= $1 - \frac{9}{n^3} + \frac{3}{n^2} - \frac{1}{n}$

`(* grafy funkci *)`

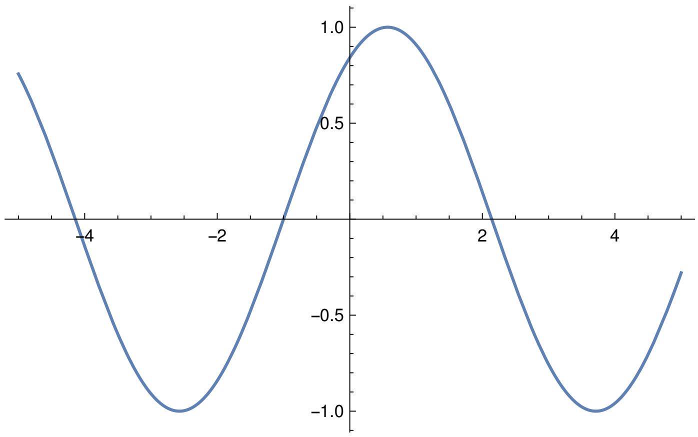
In[126]:=
```
plot1 = Plot[Sin[x], {x, -5, 5}]
```

Out[126]=



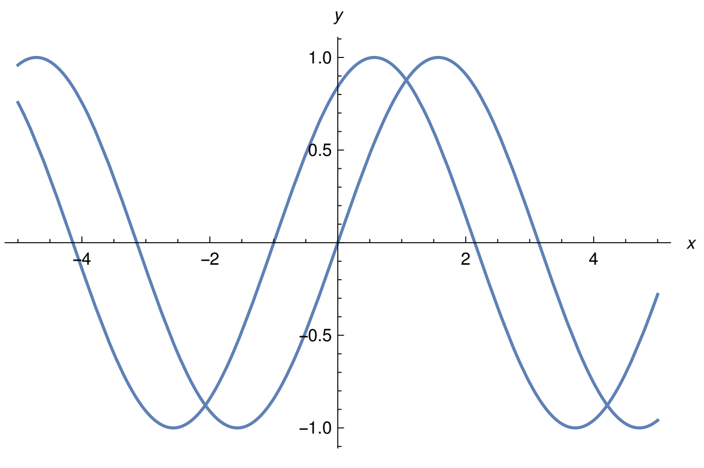
In[127]:=
```
plot2 = Plot[Sin[x + 1], {x, -5, 5}]
```

Out[127]=



In[128]:=
```
Show[{plot1, plot2}, AxesLabel → {x, y}]
```

Out[128]=



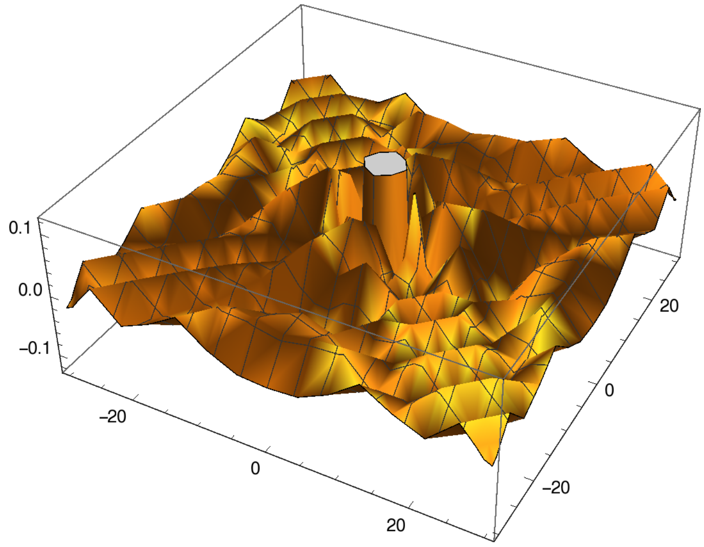
In[129]:=
```
f = Sin[Sqrt[x^2 + y^2]] / Sqrt[x^2 + y^2]
```

Out[129]=

$$\frac{\mathrm{Sin}\left[\sqrt{x^2 + y^2}\right]}{\sqrt{x^2 + y^2}}$$

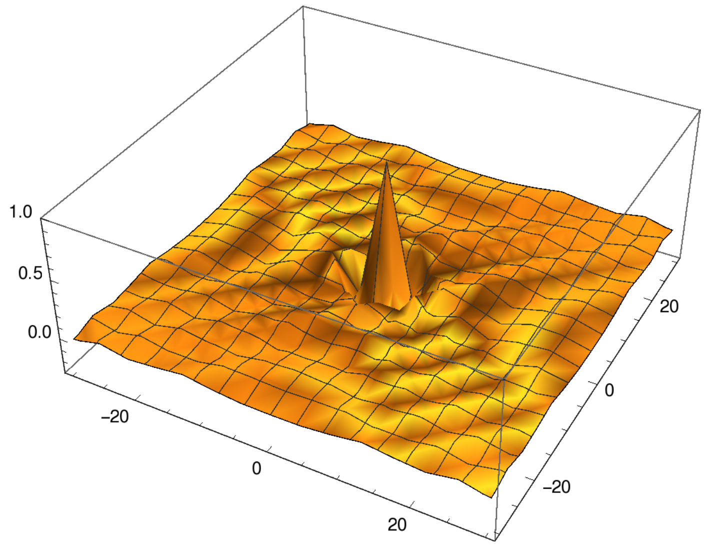
In[130]:= Plot3D[f, {x, -30, 30}, {y, -30, 30}]

Out[130]=



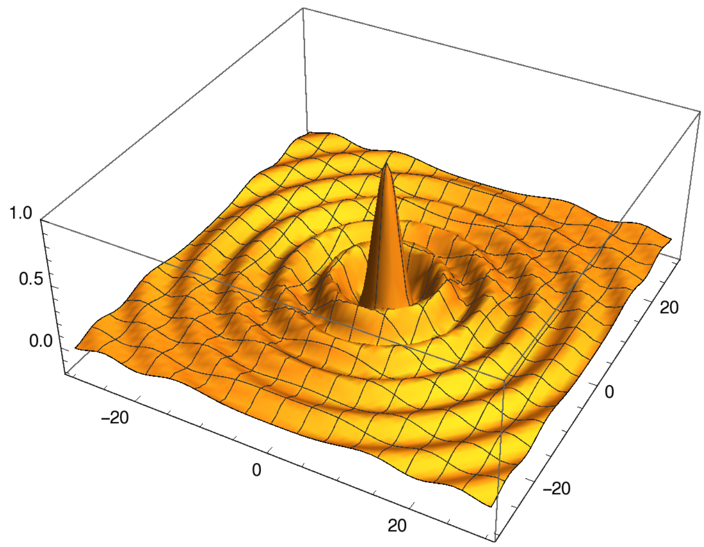
In[134]:= Plot3D[f, {x, -30, 30}, {y, -30, 30}, PlotRange → {-0.3, 1}]

Out[134]=



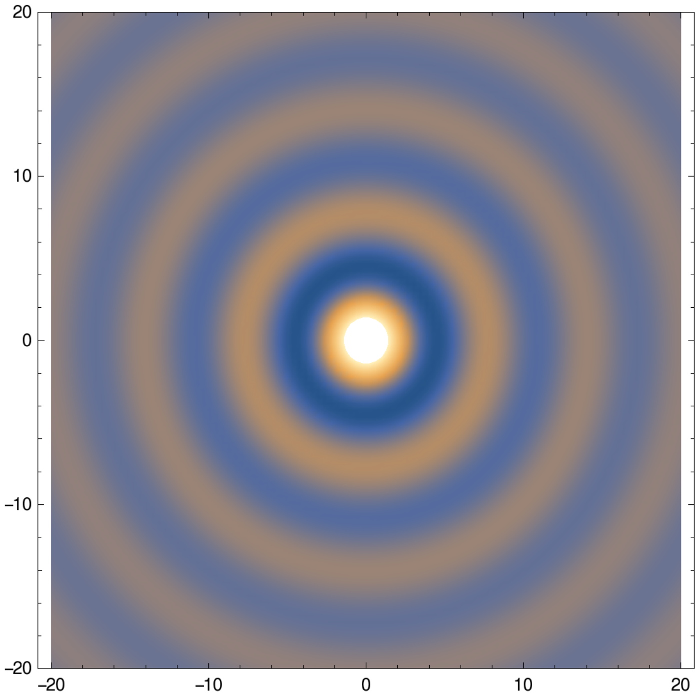
In[138]:= Plot3D[f, {x, -30, 30}, {y, -30, 30}, PlotRange → {-0.22, 1}, PlotPoints → 50]

Out[138]=



In[139]:=
```
DensityPlot[f, {x, -20, 20}, {y, -20, 20},
 PlotRange → {-0.22, 0.7}, PlotPoints → 100]
```

Out[139]=

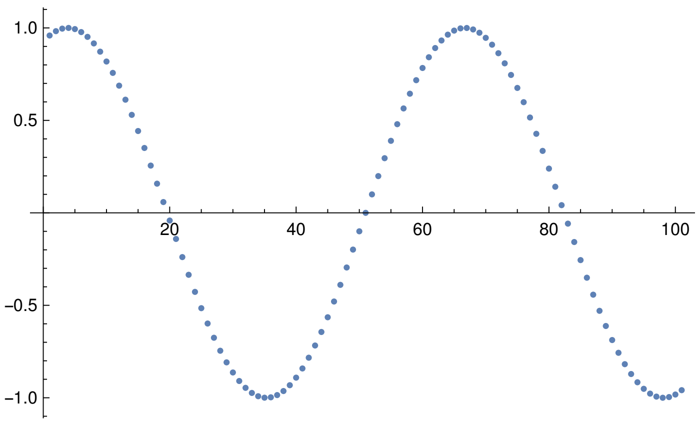



```
(* vizualizace dat *)
```

In[140]:=
```
a = Table[Sin[x], {x, -5, 5, 0.1}];
```

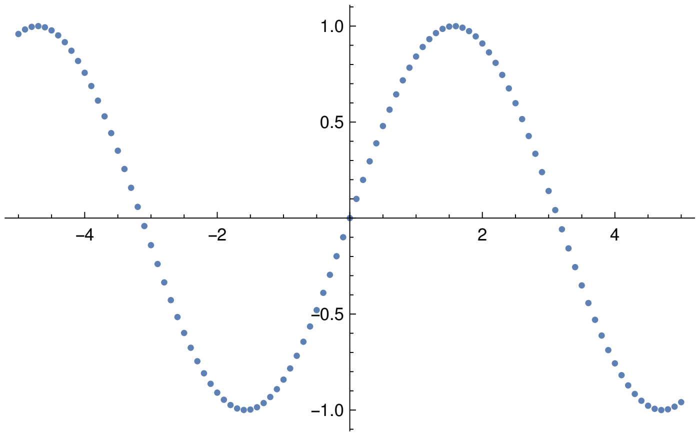
In[141]:=
```
ListPlot[a]
```

Out[141]=



In[142]:=
```
a = Table[{x, Sin[x]}, {x, -5, 5, 0.1}];
```

In[143]:= `ListPlot[a]`

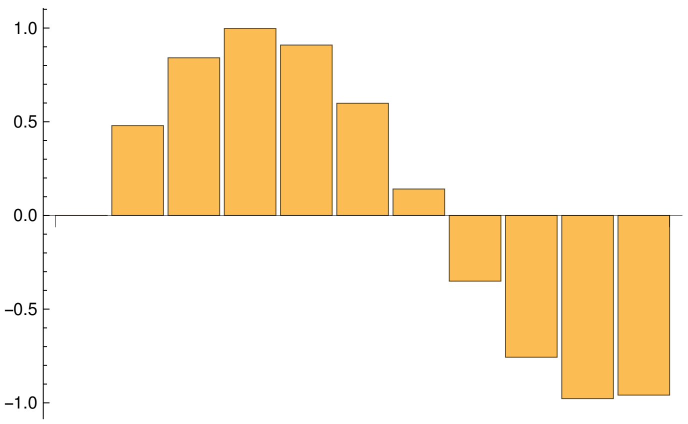
Out[143]=


In[144]:= `f`

Out[144]= $\frac{\text{Sin}\left[\sqrt{x^2 + y^2}\right]}{\sqrt{x^2 + y^2}}$

In[145]:= `a = Table[Sin[x], {x, 0, 5, 0.5}];`

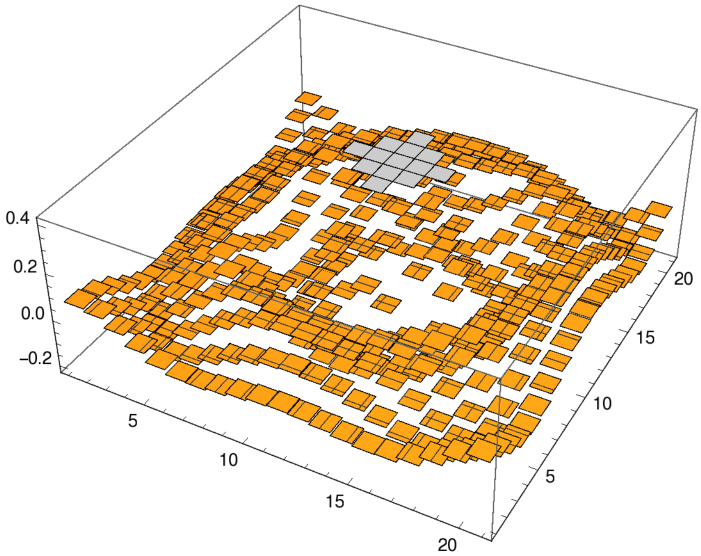
In[146]:= `BarChart[a]`

Out[146]=


In[147]:= `a = Table[f, {x, -10.001, 10}, {y, -10.001, 10}];`

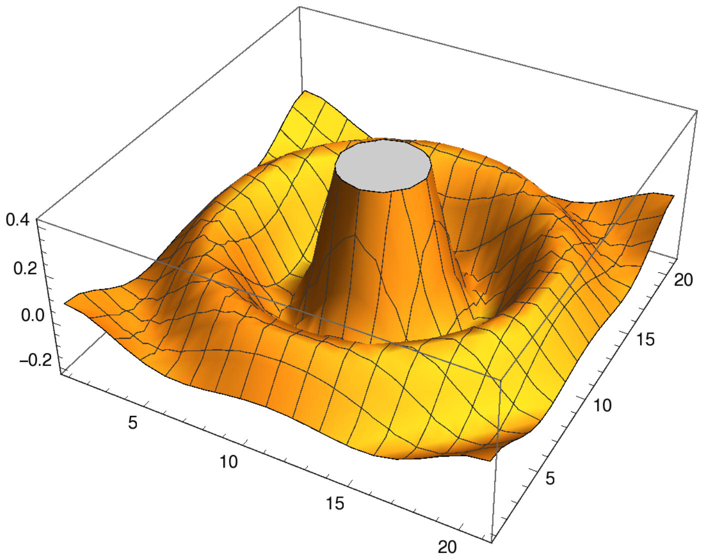
In[148]:= `ListPlot3D[a, InterpolationOrder → 0]`

Out[148]=



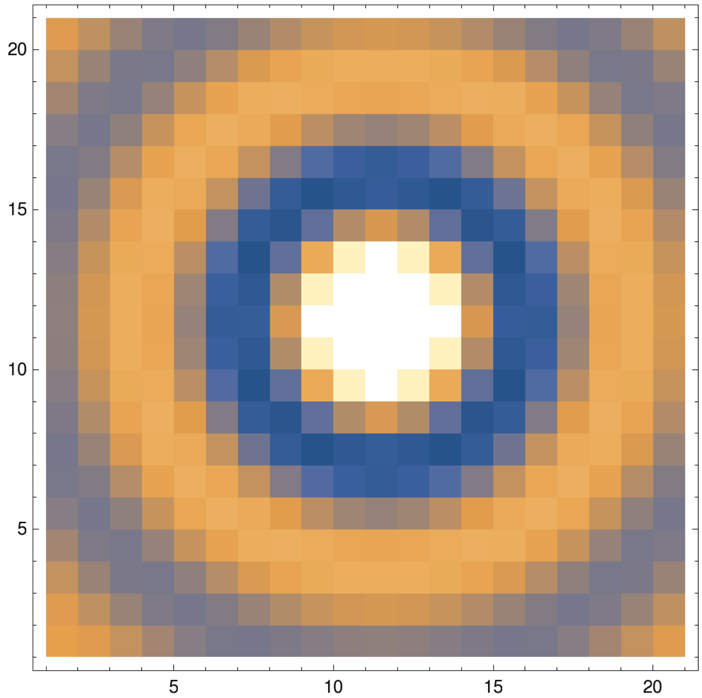
In[149]:= `ListPlot3D[a]`

Out[149]=



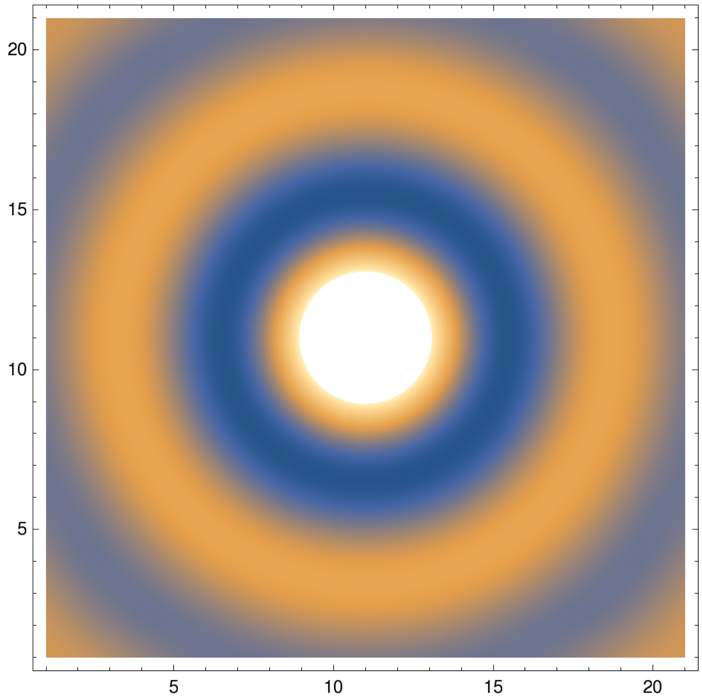
In[150]:= `ListDensityPlot[a, InterpolationOrder → 0]`

Out[150]=



In[151]:= `ListDensityPlot[a, InterpolationOrder → 3]`

Out[151]=

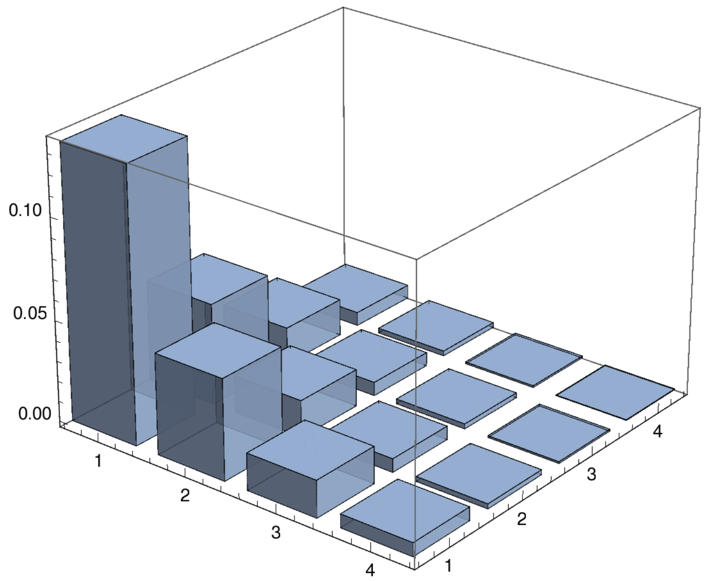


In[211]:= `a = Table[Exp[-(i + j)], {i, 1, 4}, {j, 1, 4}]`

Out[211]= $\left\{\left\{\frac{1}{e^2}, \frac{1}{e^3}, \frac{1}{e^4}, \frac{1}{e^5}\right\}, \left\{\frac{1}{e^3}, \frac{1}{e^4}, \frac{1}{e^5}, \frac{1}{e^6}\right\}, \left\{\frac{1}{e^4}, \frac{1}{e^5}, \frac{1}{e^6}, \frac{1}{e^7}\right\}, \left\{\frac{1}{e^5}, \frac{1}{e^6}, \frac{1}{e^7}, \frac{1}{e^8}\right\}\right\}$

In[286]:=
```
interp = ListInterpolation[a, {{1, 4}, {1, 4}}];
DiscretePlot3D[interp[i, j], {i, 1, 4}, {j, 1, 4}, ExtentSize → Scaled[.75],
 ViewPoint → {4, -5, 3}, BoxRatios → {1, 1, 0.7}, FillingStyle → Opacity[0.8]]
```

Out[287]=




```
(* cteni dat ze souboru *)
```

In[156]:=
```
SetDirectory["/home/rehacek/vyuka/NMP/L01"]
```

Out[156]= /home/rehacek/vyuka/NMP/L01

In[157]:=
```
data = ReadList["data.txt", Number, RecordLists → True];
```

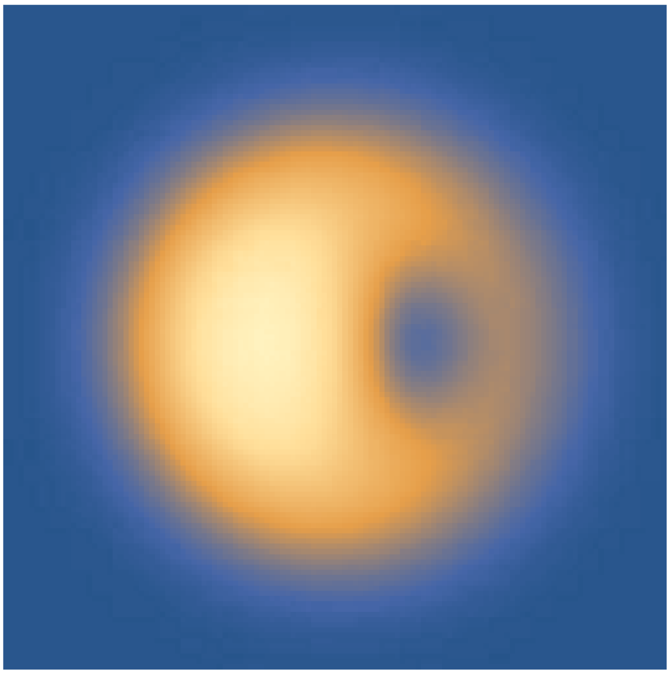
In[158]:=
```
Max[data]
```

Out[158]= 232.

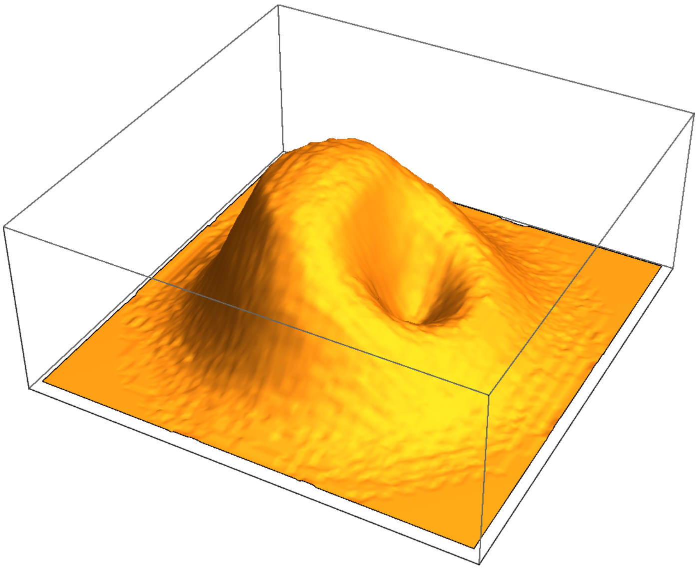
In[159]:=
```
Min[data]
```

Out[159]= 35.

In[160]:= obr1 = ListDensityPlot[data, PlotRange → All, Frame → False, InterpolationOrder → 0]

Out[160]=


In[161]:= obr2 = ListPlot3D[data, Mesh → False, Ticks → False]

Out[161]=


```
(* export grafiky do souboru *)
```

In[162]:= Export["obrazek1.jpg", obr1]

Out[162]= obrazek1.jpg

In[163]:= Export["obrazek2.jpg", obr2]

Out[163]= obrazek2.jpg

```
(* cviceni *)
```

```
(* prvocisla *)

In[164]:= prvocisla[max_] := (
   Print[2];
   For[i = 3, i ≤ max, i += 2,
    prvoc = True;
    For[del = 3, del ≤ Sqrt[i], del += 2,
      If[Divisible[i, del], prvoc = False]
     ]
     If[prvoc, Print[i]]
   ])

In[165]:= prvocisla[100]

2
3
5
7
11
13
17
19
23
29
31
37
41
43
47
53
59
61
67
71
73
79
83
89
97
```

In[166]:=
```
prvocisla[max_] := (
  list = {2};
  For[i = 3, i ≤ max, i += 2,
   prvoc = True;
   For[del = 3, del ≤ Sqrt[i], del += 2,
     If[Divisible[i, del], prvoc = False]
    ]
    If[prvoc, list = Append[list, i]]
  ];
  list)
```

In[167]:=
```
prvocisla[100]
```

Out[167]=
```
{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31,
 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97}
```

In[168]:=
```
(* statistika *)
d = ReadList["statistika.txt", Number];
```

In[169]:=
```
pocet = Length[d]
```

Out[169]=
```
10 000
```

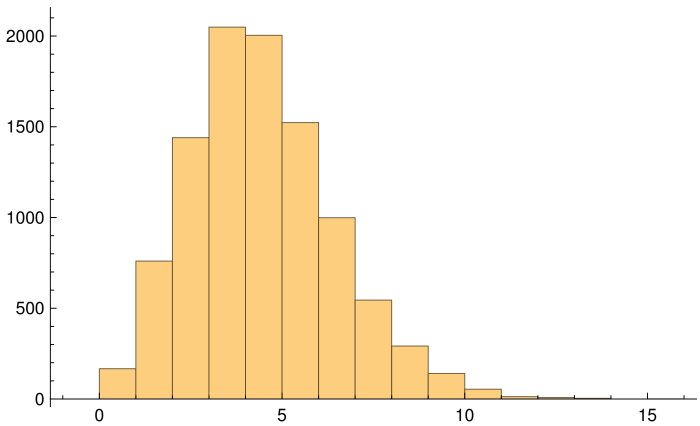
In[170]:=
```
Mean[d] // N
```

Out[170]=
```
3.9678
```

In[171]:=
```
Variance[d] // N
```

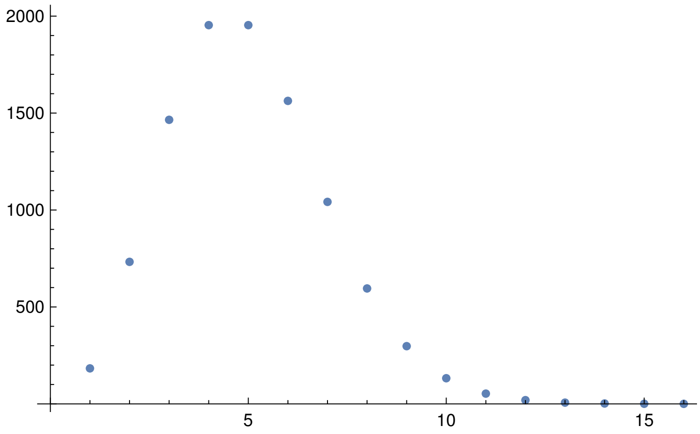
Out[171]=
```
3.94736
```

In[172]:=
```
plot1 = Histogram[d]
```

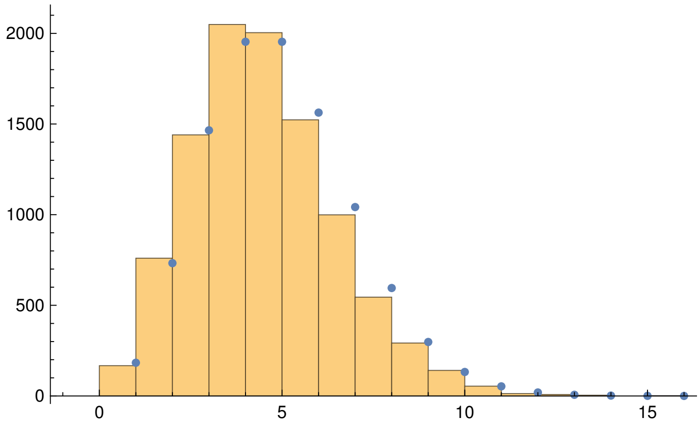
Out[172]=


In[173]:=
```
p = Table[pocet PDF[PoissonDistribution[4], i], {i, 0, 15}];
```

In[174]:=
```
plot2 = ListPlot[p]
```

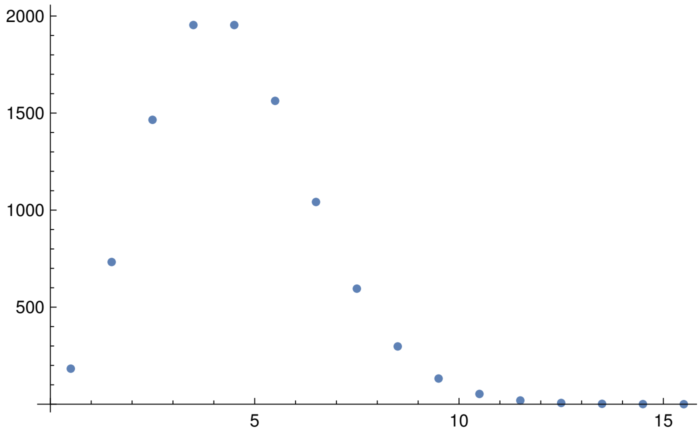
Out[174]=


In[175]:=
```
Show[{plot1, plot2}]
```

Out[175]=


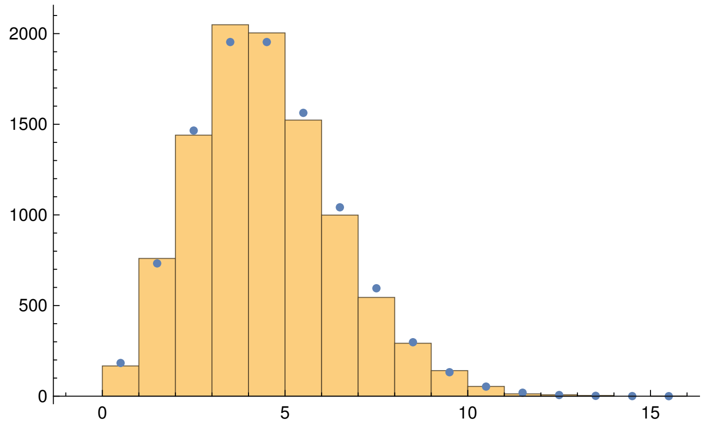
In[176]:=
```
p2 = Table[{i + 0.5, pocet PDF[PoissonDistribution[4], i]}, {i, 0, 15}];
```

In[177]:=
```
plot3 = ListPlot[p2]
```

Out[177]=


In[178]:=
```
Show[{plot1, plot3}]
```

Out[178]=




```
(* difrakce *)
```

In[179]:=
```
z =.
```

In[180]:=
```
u = Integrate[Exp[-I (x - ξ) ^2 / z], {ξ, -1, 1}] / z
```

Out[180]=

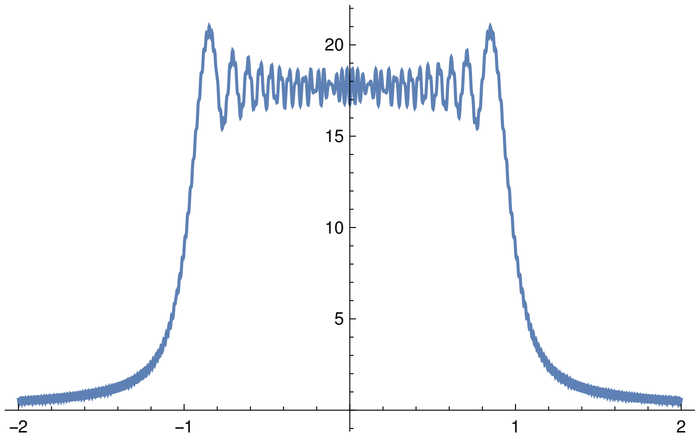
$$\frac{(-1)^{1/4}\sqrt{\pi}\left(\text{Erfi}\left[\frac{(-1)^{3/4}(-1+x)}{\sqrt{z}}\right]-\text{Erfi}\left[\frac{(-1)^{3/4}(1+x)}{\sqrt{z}}\right]\right)}{2\sqrt{z}}$$

In[181]:=
```
z = 0.01
```

Out[181]= 0.01

In[182]:=
```
Plot[Abs[u], {x, -2, 2}, AxesOrigin → {0, 0}, PlotRange → All]
```

Out[182]=

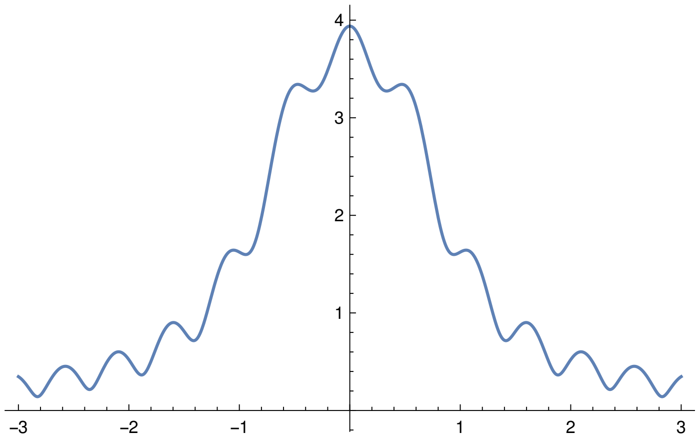


In[183]:=
```
z = 0.3
```

Out[183]= 0.3

In[184]:=
```
Plot[Abs[u], {x, -10 z, 10 z}, AxesOrigin → {0, 0}, PlotRange → All]
```

Out[184]=

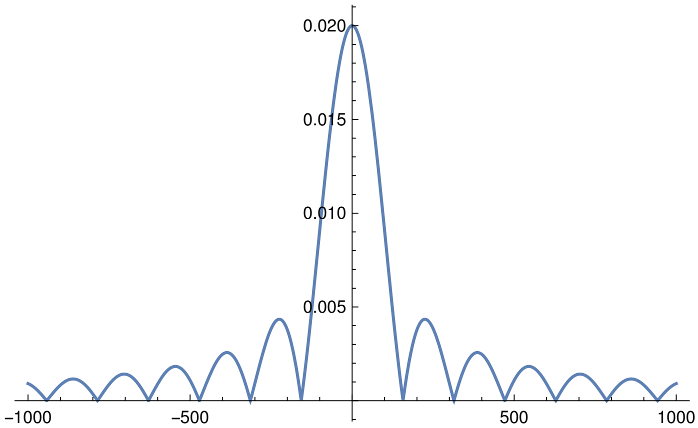


In[185]:=
```
z = 100
```

Out[185]= 100

In[186]:=
```
Plot[Abs[u], {x, -10 z, 10 z}, AxesOrigin → {0, 0}, PlotRange → All]
```

Out[186]=



In[187]:=
```
z =.
```

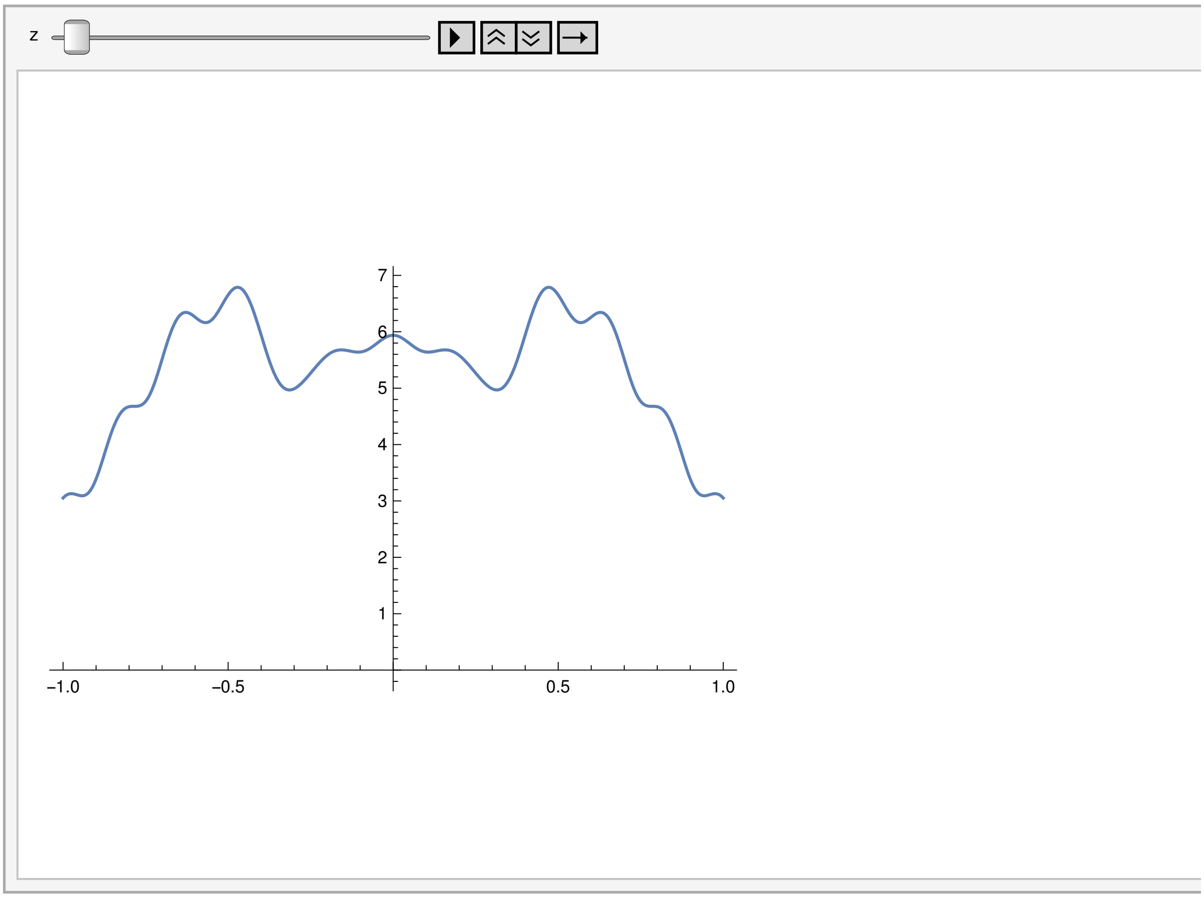
In[188]:=
```
u
```

Out[188]=
$$\frac{(-1)^{1/4}\sqrt{\pi}\left(\text{Erfi}\left[\frac{(-1)^{3/4}(-1+x)}{\sqrt{z}}\right]-\text{Erfi}\left[\frac{(-1)^{3/4}(1+x)}{\sqrt{z}}\right]\right)}{2\sqrt{z}}$$

In[189]:=
$$\text{fresnel}[x\_, z\_] := \left(\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{\frac{\pi}{2}}\left(\text{Erf}\left[\frac{(-1)^{1/4}(-1+x)}{\sqrt{z}}\right]-\text{Erf}\left[\frac{(-1)^{1/4}(1+x)}{\sqrt{z}}\right]\right)}{\sqrt{z}}\right)$$

In[190]:=
```
Animate[Plot[Abs[fresnel[x, z]], {x, -10 z, 10 z}, AxesOrigin → {0, 0},
  PlotRange → All], {z, 0.1, 1}, AnimationRunning → False]
```

Out[190]=



```
(* Wienuv posunovaci zakon *)
(* f=2πhc^2/λ^5/(Exp[hc/(λkT)]-1) *)
```

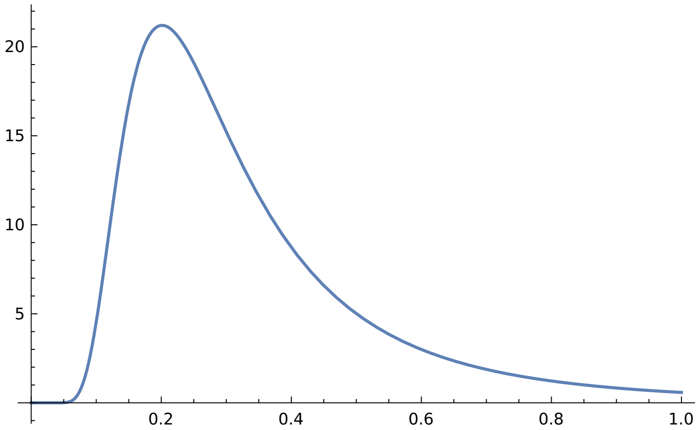
In[•]:=
```
b =.; x =.
```

In[•]:=
```
f = 1 / λ ^ 5 / (Exp[b / λ] - 1)
```

Out[•]= $\dfrac{1}{\left(-1+e^{b/\lambda}\right)\lambda^5}$

In[•]:=
```
b = 1; Plot[f, {λ, 0, 1}, PlotRange → All]
```

General: $2.81067\times10^{23}\ 6.7539397714\times10^{-21260}$ is too small to represent as a normalized machine number; precision may be lost.

Out[•]=


In[•]:=
```
b =.
```

In[•]:=
```
eq = Simplify[D[f, λ] == 0]
```

Out[•]= $5+\dfrac{b\,e^{b/\lambda}}{\lambda-e^{b/\lambda}\lambda}$ == 0

In[•]:=
```
lhs = Factor[eq[[1]]]
```

Out[•]= $-\dfrac{b\,e^{b/\lambda}+5\lambda-5\,e^{b/\lambda}\lambda}{\left(-1+e^{b/\lambda}\right)\lambda}$

In[•]:=
```
cit = Numerator[lhs]
```

Out[•]= $-b\,e^{b/\lambda}-5\lambda+5\,e^{b/\lambda}\lambda$

In[•]:=
```
cit2 = Expand[cit / λ]
```

Out[•]= $-5+5\,e^{b/\lambda}-\dfrac{b\,e^{b/\lambda}}{\lambda}$

In[•]:=
```
b = λ x
```

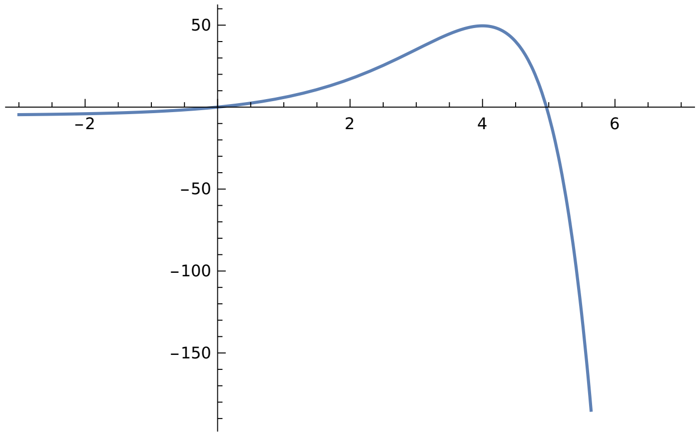
Out[•]= $x\,\lambda$

In[•]:=
```
cit3 = Simplify[cit2]
```

Out[•]= $-5-e^{x}(-5+x)$

In[ ]:=
```
Plot[cit3, {x, -3, 7}]
```

Out[ ]=



In[ ]:=
```
koren = FindRoot[cit3, {x, 1}]
```

Out[ ]= $\{x \to 3.08169 \times 10^{-16}\}$

In[ ]:=
```
koren = FindRoot[cit3, {x, 7}]
```

Out[ ]= $\{x \to 4.96511\}$

In[ ]:=
```
hc / k == koren[[1, 2]] λ T
```

Out[ ]= $\frac{hc}{k} == 4.96511\, T\, \lambda$

```
(* MTF kruhova apertura *)
```

In[ ]:=
```
prekryv = Integrate[Integrate[1, {y, 0, Sqrt[1 - (x + a / 2) ^ 2]}],
   {x, 0, 1 - a / 2}, Assumptions → 0 < a < 1]
```

Out[ ]= $\frac{1}{8}\left(-a\sqrt{4-a^2} + 4\,\text{ArcCos}\left[\frac{a}{2}\right]\right)$

In[ ]:=
```
norma = Simplify[prekryv, a == 0]
```

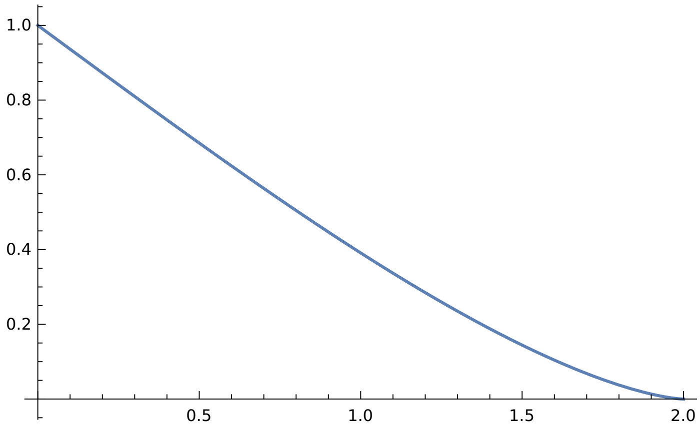
Out[ ]= $\frac{\pi}{4}$

In[ ]:=
```
otf = Simplify[prekryv / norma]
```

Out[ ]= $-\frac{a\sqrt{4-a^2} - 4\,\text{ArcCos}\left[\frac{a}{2}\right]}{2\pi}$

In[ ]:=
```
Plot[prekryv / norma, {a, 0, 2}]
```

Out[ ]=



```
(* rektangularni apertura + defokus *)
```

In[ ]:=
```
a =.; e =.
```

In[ ]:=
```
prekryv = Re[Integrate[Exp[-I e (x - a / 2)^2] Exp[I e (x + a / 2)^2], {x, 0, (1 - a) / 2}]]
```

Out[ ]= $\frac{1}{2}\,\text{Im}\left[\frac{-1+e^{-i\,(-1+a)\,a\,e}}{a\,e}\right]$

In[ ]:=
```
norma = prekryv /. a → 0
```

Power: Infinite expression $\frac{1}{0}$ encountered.

Infinity: Indeterminate expression $\frac{0\,\text{ComplexInfinity}}{e}$ encountered.

Out[ ]= Indeterminate

In[ ]:=
```
norma = Limit[prekryv, a → 0]
```

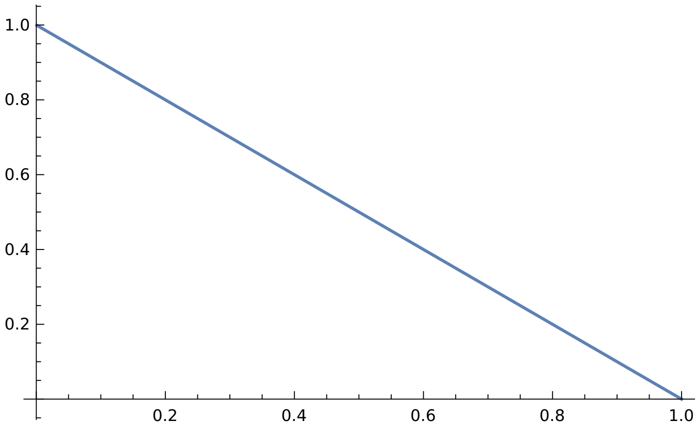
Out[ ]= $\frac{1}{2}$

In[ ]:=
```
otf = prekryv / norma
```

Out[ ]= $\text{Im}\left[\frac{-1+e^{-i\,(-1+a)\,a\,e}}{a\,e}\right]$

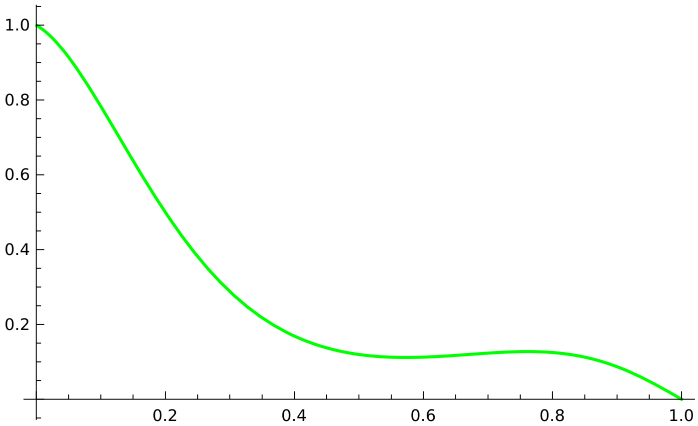
In[ ]:= pl1 = Plot[otf /. e → 0.001, {a, 0.001, 1}, PlotRange → All]

Out[ ]=



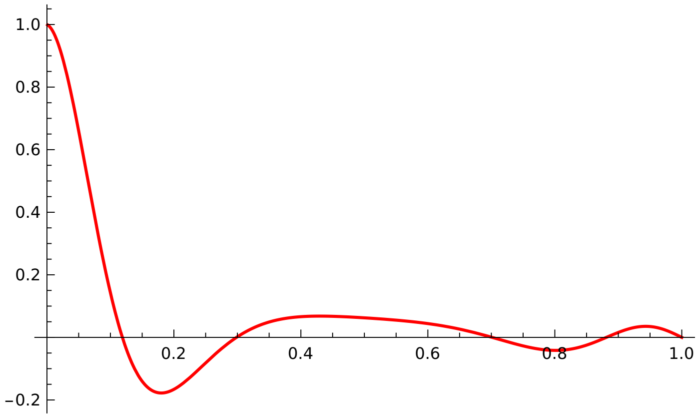
In[ ]:= pl2 = Plot[otf /. e → 10, {a, 0.001, 1}, PlotRange → All, PlotStyle → Green]

Out[ ]=



In[ ]:= pl3 = Plot[otf /. e → 30, {a, 0.001, 1}, PlotRange → All, PlotStyle → Red]

Out[ ]=



In[ ]:= **Show[{pl1, pl2, pl3}, PlotRange → All]**

Out[ ]=