splajn

$$A = \frac{x_{j+1} - x}{x_{j+1} - x_j}$$

$$\dot{A} = -\frac{1}{x_{j+1} - x_j} \quad , \quad \ddot{A} = 0$$

$$C = \frac{1}{6}(A^3 - A)(x_{j+1} - x_j)^2$$

$$\dot{C} = \frac{1}{6}(x_{j+1} - x_j)^2 (3A^2\dot{A} - \dot{A})$$

$$\ddot{C} = \frac{1}{6}(x_{j+1} - x_j)^2 (6A\dot{A}^2)$$

$$\ddot{C} = A$$

```
In[1]:= (* konstrukce splajnu pro 3 body *)

In[2]:= (* vzorky *)

In[3]:= x1 = 1; x2 = 2; x3 = 3
Out[3]= 3

In[4]:= y1 = 4; y2 = 1; y3 = 2
Out[4]= 2

In[5]:= (* druhe derivace (pocatecni volba) *)

In[6]:= yy1 = 10; yy2 = 5; yy3 = 0
Out[6]= 0

In[7]:= (* koeficienty linearni interpolace *)

In[8]:= a = (x2 - x) / (x2 - x1)
Out[8]= 2 - x

In[9]:= b = 1 - a
Out[9]= -1 + x

In[10]:= aa = (x3 - x) / (x3 - x2)
Out[10]= 3 - x

In[11]:= bb = 1 - aa
Out[11]= -2 + x

In[12]:= (* koeficienty kubicke interpolace *)

In[13]:= c = 1/6 (a^3 - a) (x2 - x1)^2
```

Out[13]= $\frac{1}{6}\left(-2+(2-x)^3+x\right)$

```
In[14]:= d = 1/6 (b^3 - b) (x2 - x1)^2
```

Out[14]= $\frac{1}{6}\left(1+(-1+x)^3-x\right)$

```
In[15]:= cc = 1/6 (aa^3 - aa) (x3 - x2)^2
```

Out[15]= $\frac{1}{6}\left(-3+(3-x)^3+x\right)$

```
In[16]:= dd = 1/6 (bb^3 - bb) (x3 - x2)^2
```

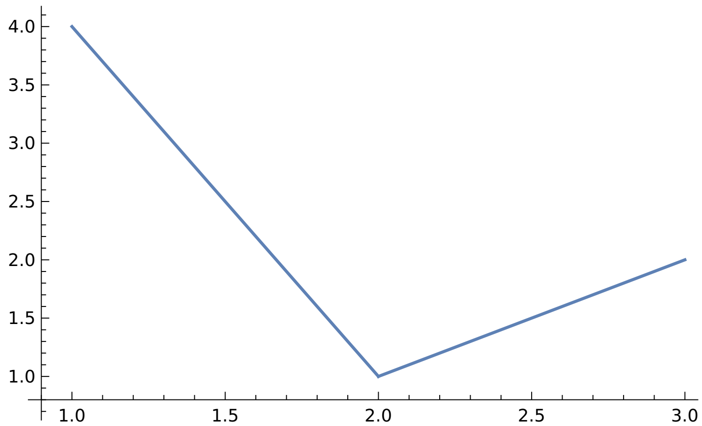
Out[16]= $\frac{1}{6}\left(2+(-2+x)^3-x\right)$

```
In[17]:= (* linearni interpolacni funkce *)
```

In[18]:=
```
f = a y1 + b y2
```
Out[18]= $-1 + 4\,(2 - x) + x$

In[19]:=
```
ff = aa y2 + bb y3
```
Out[19]= $3 + 2\,(-2 + x) - x$

In[20]:=
```
p1 = Plot[f, {x, x1, x2}];
p2 = Plot[ff, {x, x2, x3}];
Show[{p1, p2}, PlotRange → All, AxesOrigin → {0.9, 0.8}]
```
Out[20]=



In[21]:=
```
(* kubicke interpolace s navazujici 2 derivaci *)
```

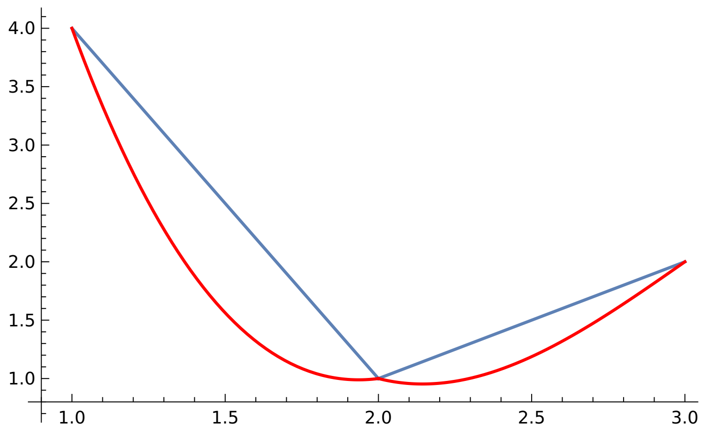
In[22]:=
```
g = f + c yy1 + d yy2
```
Out[22]= $-1 + 4\,(2 - x) + \frac{5}{6}\left(1 + (-1 + x)^3 - x\right) + x + \frac{5}{3}\left(-2 + (2 - x)^3 + x\right)$

In[23]:=
```
gg = ff + cc yy2 + dd yy3
```
Out[23]= $3 + 2\,(-2 + x) - x + \frac{5}{6}\left(-3 + (3 - x)^3 + x\right)$

In[24]:=
```
(* srovnani linearni a kubicke interpolace *)
```

In[25]:=
```
p3 = Plot[g, {x, x1, x2}, PlotStyle → Red];
p4 = Plot[gg, {x, x2, x3}, PlotStyle → Red];
Show[{p1, p2, p3, p4}, PlotRange → All, AxesOrigin → {0.9, 0.8}]
```

Out[25]=



In[26]:=
```
(* druhe derivace interpolacni funkce *)
```

In[27]:=
```
der2g = D[g, x, x]
```

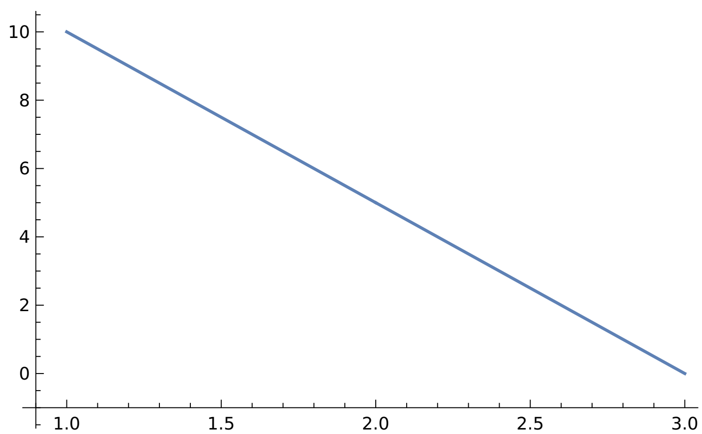
Out[27]= $10\,(2-x)+5\,(-1+x)$

In[28]:=
```
der2gg = D[gg, x, x]
```

Out[28]= $5\,(3-x)$

In[29]:=
```
p5 = Plot[der2g, {x, x1, x2}];
p6 = Plot[der2gg, {x, x2, x3}];
Show[p5, p6, PlotRange → All, AxesOrigin → {0.9, -1}]
```

Out[29]=



In[30]:=
```
(* prvni derivace interpolacni funkce *)
```

In[31]:=
```
der1g = D[g, x]
```

Out[31]= $-3+\frac{5}{3}\left(1-3\,(2-x)^2\right)+\frac{5}{6}\left(-1+3\,(-1+x)^2\right)$

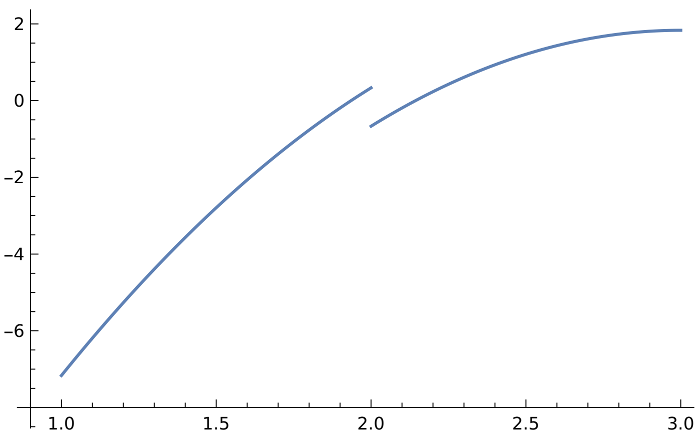
In[32]:=
```
der1gg = D[gg, x]
```

Out[32]= $1 + \frac{5}{6} \left(1 - 3 \, (3 - x)^2\right)$

In[33]:=
```
(* nespojitost prvni derivace ve vnitrnim bode *)
```

In[34]:=
```
p7 = Plot[der1g, {x, x1, x2}];
p8 = Plot[der1gg, {x, x2, x3}];
Show[p7, p8, PlotRange → All, AxesOrigin → {0.9, -8}]
```

Out[34]=



In[35]:=
```
(* konstrukce splajnu: druha der. v x2 bude stupen volnosti *)
```

In[36]:=
```
newderg = D[f + c yy1 + d der2y2, x]
```

Out[36]= $-3 + \frac{5}{3} \left(1 - 3 \, (2 - x)^2\right) + \frac{1}{6} \, \text{der2y2} \left(-1 + 3 \, (-1 + x)^2\right)$

In[37]:=
```
newdergg = D[ff + cc der2y2 + dd yy3, x]
```

Out[37]= $1 + \frac{1}{6} \, \text{der2y2} \left(1 - 3 \, (3 - x)^2\right)$

In[38]:=
```
(* pozaduji navazujici prvni derivace v x2 *)
```

In[39]:=
```
Solve[(newderg /. x → x2) == (newdergg /. x → x2), der2y2]
```

Out[39]= $\left\{\left\{\text{der2y2} \to \frac{7}{2}\right\}\right\}$

In[40]:=
```
(* nova druha derivace v bode x2 *)
```

In[41]:=
```
yy2 = 7 / 2
```

Out[41]= $\frac{7}{2}$

In[42]:=
```
(* opakuji kubickou interpolaci s touto novou hodnotou *)
```

In[43]:=
```
g = f + c yy1 + d yy2
```

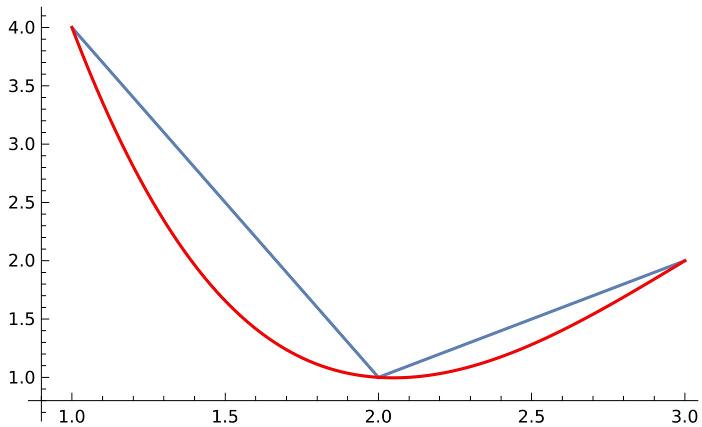
Out[43]= $-1 + 4 \, (2 - x) + \frac{7}{12} \left(1 + (-1 + x)^3 - x\right) + x + \frac{5}{3} \left(-2 + (2 - x)^3 + x\right)$

In[44]:=
```
gg = ff + cc yy2 + dd yy3
```

Out[44]= $3+2\,(-2+x)-x+\frac{7}{12}\left(-3+(3-x)^3+x\right)$

In[45]:=
```
p3 = Plot[g, {x, x1, x2}, PlotStyle → Red];
p4 = Plot[gg, {x, x2, x3}, PlotStyle → Red];
Show[{p1, p2, p3, p4}, PlotRange → All, AxesOrigin → {0.9, 0.8}]
```

Out[45]=



In[46]:=
```
der2g = D[g, x, x]
```

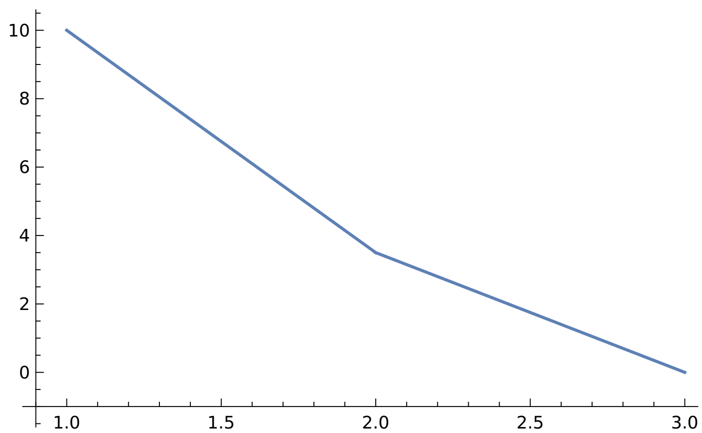
Out[46]= $10\,(2-x)+\frac{7}{2}\,(-1+x)$

In[47]:=
```
der2gg = D[gg, x, x]
```

Out[47]= $\frac{7\,(3-x)}{2}$

In[48]:=
```
p5 = Plot[der2g, {x, x1, x2}];
p6 = Plot[der2gg, {x, x2, x3}];
Show[p5, p6, PlotRange → All, AxesOrigin → {0.9, -1}]
```

Out[48]=



In[49]:= **der1g = D[g, x]**

Out[49]= $-3+\frac{5}{3}\left(1-3\,(2-x)^2\right)+\frac{7}{12}\left(-1+3\,(-1+x)^2\right)$

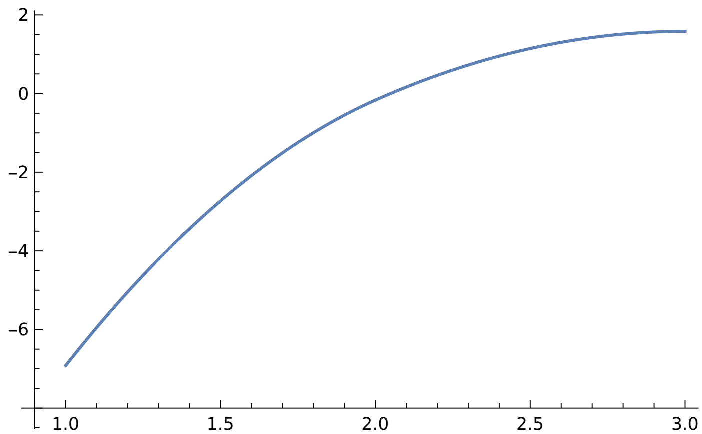
In[50]:= **der1gg = D[gg, x]**

Out[50]= $1+\frac{7}{12}\left(1-3\,(3-x)^2\right)$

In[51]:= **(* nespojitost prvni derivace je odstranena *)**

In[52]:=
```
p7 = Plot[der1g, {x, x1, x2}];
p8 = Plot[der1gg, {x, x2, x3}];
Show[p7, p8, PlotRange → All, AxesOrigin → {0.9, -8}]
```

Out[52]=



```
(* trigonometricka interpolace pro 3 body *)
```

```
In[ ]:= dim = 3
Out[ ]= 3
```

```
In[ ]:= x = Table[2 π n / dim, {n, 0, dim - 1}]
```

Out[ ]= $\left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

```
In[ ]:= (* vzorkovane funkcni hodnoty *)
```

```
In[ ]:= y = RandomReal[1, dim]
Out[ ]= {0.0360832, 0.632988, 0.159449}
```

```
In[ ]:= (* DFT koeficienty *)
```

```
In[ ]:= ak = Table[Sum[y[[n + 1]] Exp[-I 2 π n k / dim], {n, 0, dim - 1}], {k, 0, dim - 1}]
Out[ ]= {0.828521, -0.360136 - 0.410097 i, -0.360136 + 0.410097 i}
```

```
In[ ]:= (* interpolace inverzni DFT *)
```

```
In[ ]:= interp = 1 / dim Sum[ak[[k + 1]] Exp[I xx k], {k, 0, dim - 1}]
```

Out[ ]= $\frac{1}{3}\left(0.828521 - (0.360136 + 0.410097\, i)\, e^{i\, xx} - (0.360136 - 0.410097\, i)\, e^{2\, i\, xx}\right)$

```
In[ ]:= (* overeni, ze interpolace prochazi prvnim vzorkem *)
```

```
In[ ]:= Simplify[interp //. xx → 0]
Out[ ]= 0.0360832 + 0. i
```

```
In[ ]:= (* graf interpolacni funkce se zakreslenymi vzorky *)
```

```
In[ ]:= p1 = ListPlot[Transpose[{x, y}], PlotStyle → Red];
p2 = Plot[Re[interp], {xx, 0, 6 π}];
Show[{p1, p2}, PlotRange -> {0, 1}, AxesLabel → {"x", "y"}]
```

Out[ ]=