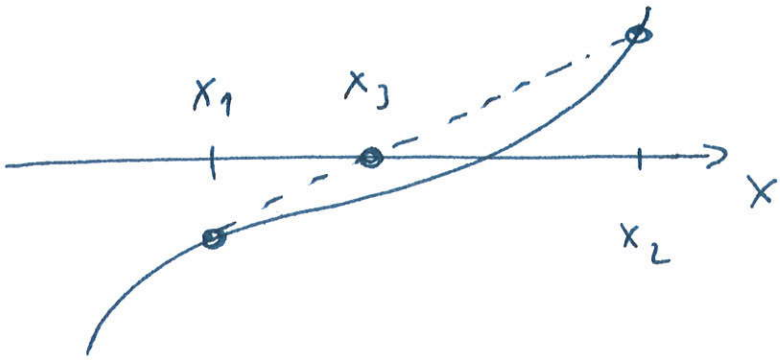
sečny – lineární interpolace



$$\frac{x_2 - x_3}{f(x_2) - 0} = \frac{x_2 - x_1}{f(x_2) - f(x_1)}$$

$$x_2 - x_3 = f(x_2)\,\frac{x_2 - x_1}{f(x_2) - f(x_1)}$$

$$x_3 = x_2 - f(x_2)\,\frac{x_2 - x_1}{f(x_2) - f(x_1)}$$

Ridders

$x_1, x_2, x_3 = \frac{x_1 + x_2}{2}$

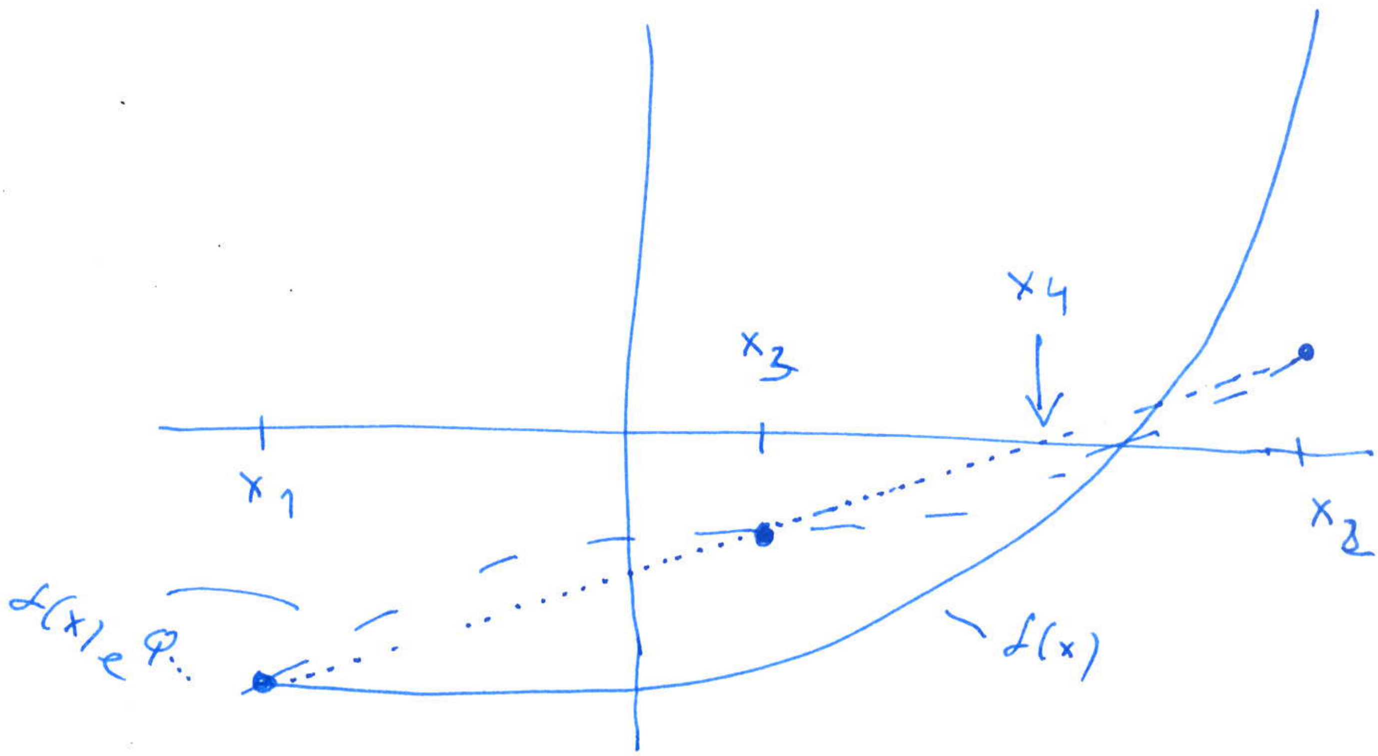
odstranění ohybu

$$f(x) \to f(x)\, e^{Q \frac{x - x_1}{x_3 - x_1}} \quad \text{(zachová kořen)}$$

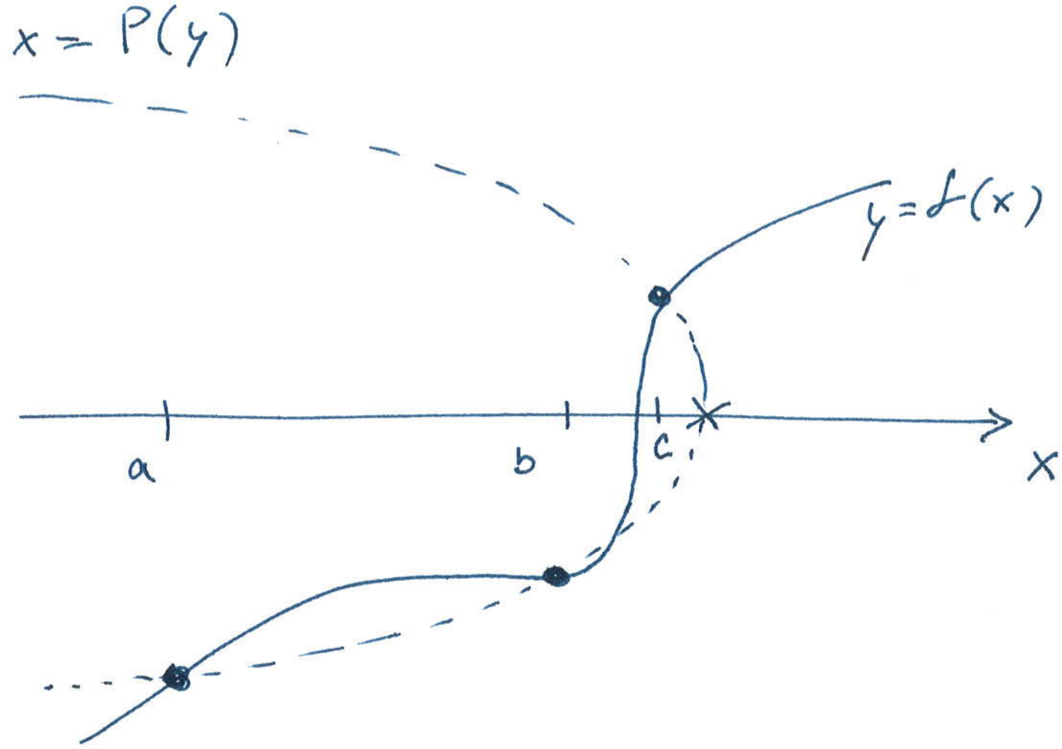
tj. $f(x_1)$, $f(x_2) \to f(x_2)\, e^{2Q}$, $f(x_3) \to f(x_3)\, e^{Q}$

linearizace

$$\frac{f(x_1) + f(x_2)\, e^{2Q}}{2} = f(x_3)\, e^{Q}$$



Brent – inv. kvadr. interpolace



Konvergence Newton-Raphson

rozvoj $f(x)$ kolem kořene $f(x_0)=0$

$$f(x_0+\epsilon) = 0 + \epsilon f'(x_0) + \epsilon^2 \frac{f''(x_0)}{2}$$

$$f'(x_0+\epsilon) = f'(x_0) + \epsilon f''(x_0)$$

—

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$ n-tá iterace N-R

$$x_n = x_0 + \epsilon_n$$

$$x_{n+1} = x_0 + \epsilon_{n+1}$$

—

$$\epsilon_{n+1} = \epsilon_n - \frac{f(x_n)}{f'(x_n)} = \epsilon_n - \frac{\epsilon_n f'(x_0) + \epsilon_n^2 \frac{f''(x_0)}{2}}{f'(x_0)\left[1+\epsilon_n \frac{f''(x_0)}{f'(x_0)}\right]}$$

$$\simeq \epsilon_n - \left[\epsilon_n + \epsilon_n^2 \frac{f''(x_0)}{2f'(x_0)}\right]\left[1 - \epsilon_n \frac{f''(x_0)}{f'(x_0)}\right]$$

$$\simeq \epsilon_n^2 \frac{f''(x_0)}{2f'(x_0)}$$

Newton-Raphson (soustavy)

$$f_i(x_1+\delta x_1, x_2+\delta x_2, \dots, x_N+\delta x_N)$$

$$\cong f_i(x_1, x_2, \dots, x_N) + \sum_j \underbrace{\frac{\partial f_i(x_1, x_2, \dots, x_N)}{\partial x_j}}_{J_{ij} \equiv \frac{\partial f_i}{\partial x_j}} \delta x_j$$

$$f_i(\vec{x}+\vec{\delta_x}) = f_i(\vec{x}) + \sum_j J_{ij}(\vec{x})\, \delta x_j$$

$$= f_i(\vec{x}) + (J\, \vec{\delta_x})_i$$

$$\vec{f}(\vec{x}+\vec{\delta_x}) = \vec{f}(\vec{x}) + J(\vec{x})\, \vec{\delta_x}$$