# PSF / Strehl

(L7)

$$I_N(x', y') = \frac{|a(x', y')|^2}{|a(0,0)|^2}, \quad a(x', y') = \mathcal{F}\{P(x_p, y_p)\}$$

## Strehl

$$D = \frac{|a(0,0)|^2}{|a(0,0)|^2_{W=0}} = \frac{\left|\iint P_0(x_p, y_p)\, e^{ikW(x_p, y_p)}\, dx_p\, dy_p\right|^2}{\left|\iint P_0(x_p, y_p)\, dx_p\, dy_p\right|^2}$$

$$P_0(x_p, y_p) \begin{cases} 1 & \text{uvnitř } S_p \\ 0 & \text{vně } S_p \end{cases} \Rightarrow \iint_{S_p} dx_p\, dy_p = S_p$$

$$D = \left|\frac{\iint_{S_p} e^{ikW}\, dx_p\, dy_p}{S_p}\right|^2 = \left|\langle e^{ikW}\rangle\right|^2$$

## Kruh. pupila

$$\iint_{S_p} dx_p\, dy_p = \int_0^1 \int_0^{2\pi} R_p\, dR_p\, d\varphi_p = \pi$$

$$D = \left|\frac{1}{\pi}\int_0^1 \int_0^{2\pi} e^{ikW(R_p, \varphi_p)} R_p\, dR_p\, d\varphi_p\right|^2$$

malé deformace vlnopl.

$$e^{ikW} \approx 1 + ikW - \frac{(kW)^2}{2} + \dots$$

$$D \approx \left| 1 + ik \underbrace{\frac{1}{\pi}\iint W R_p \, dR_p \, d\varphi_p}_{\langle W \rangle} - \frac{k^2}{2} \underbrace{\frac{1}{\pi}\iint W^2 R_p \, dR_p \, d\varphi_p}_{\langle W^2 \rangle} \right|^2$$

$$= \left| 1 - \frac{k^2}{2}\langle W^2 \rangle + ik\langle W \rangle \right|^2$$

$$= \left( 1 - \frac{k^2}{2}\langle W^2 \rangle \right)^2 + k^2 \langle W \rangle^2$$

$$D \approx 1 - k^2 \underbrace{\left( \langle W^2 \rangle - \langle W \rangle^2 \right)}_{\Delta W^2}$$

# rozostření

$$W_{020} = A_{020} R_p^2 , \quad A_{020} = -u^2 \frac{\Delta z'}{2}$$

$$D = 1 - k^2 \left( \langle W^2 \rangle - \langle W \rangle^2 \right)$$

$$\langle W \rangle = A_{020} \underbrace{\frac{1}{\pi} \iint R_p^2 R_p \, dR_p \, d\varphi_p}_{\langle R_p^2 \rangle = \frac{1}{\pi} 2\pi \left. \frac{R_p^4}{4} \right|_0^1 = \frac{1}{2}} = \frac{A_{020}}{2}$$

$$\langle W^2 \rangle = A_{020}^2 \underbrace{\frac{1}{\pi} \iint R_p^4 R_p \, dR_p \, d\varphi_p}_{\langle R_p^4 \rangle = \frac{1}{\pi} 2\pi \left. \frac{R_p^6}{6} \right|_0^1 = \frac{1}{3}} = \frac{A_{020}^2}{3}$$

$$\langle W^2 \rangle - \langle W \rangle^2 = \frac{A_{020}^2}{3} - \frac{A_{020}^2}{4} = \frac{A_{020}^2}{12}$$

$$D = 1 - \frac{1}{12} k^2 A_{020}^2$$

užitečné vztahy: $\langle R_p^2 \rangle = \frac{1}{2}$ , $\langle R_p^4 \rangle = \frac{1}{3}$ !

$$\langle R_p^n \rangle = \frac{2}{n+2}$$

přípustné rozostření

$$D_{min} = 0{,}8$$

$$1 - \frac{1}{12}\xi^2\left(-\frac{u^2 \Delta z'}{2}\right)^2 = 0{,}8$$

$$\frac{1}{12}\xi^2 \frac{u^4 \Delta z'^2}{4} = 0{,}2$$

$$\frac{1}{12}\frac{4\pi^2}{\lambda^2}\frac{u^4 \Delta z'^2}{4} = 0{,}2$$

$$\Delta z' = \sqrt{2{,}4\frac{\lambda^2}{u^4 \pi^2}} = \sqrt{2{,}4}\,\frac{\lambda}{u^2 \pi}$$

$$\sqrt{2{,}4} \sim 1{,}55 \sim \frac{\pi}{2}$$

$$\boxed{\Delta z' \approx \pm \frac{\lambda}{2u^2}}$$

4x menší než maximální rozostření !

# Optimální zaostření

$$W = W_s + A_{020} R_\rho^2$$

$$\langle W^2 \rangle = \langle W_s^2 \rangle + 2 A_{020} \langle W_s R_\rho^2 \rangle + A_{020}^2 \underbrace{\langle R_\rho^4 \rangle}_{1/3}$$

$$\langle W \rangle^2 = \left( \langle W_s \rangle + A_{020} \underbrace{\langle R_\rho^2 \rangle}_{1/2} \right)^2 = \langle W_s \rangle^2 + A_{020} \langle W_s \rangle + \frac{A_{020}^2}{4}$$

$$\Delta W^2 = \Delta W_s^2 + A_{020} \left( 2 \langle W_s R_\rho^2 \rangle - \langle W_s \rangle \right)$$

$$+ A_{020}^2 \frac{1}{12}$$

$$\frac{\partial D}{\partial A_{020}} = - k^2 \left[ 2 \langle W_s R_\rho^2 \rangle - \langle W_s \rangle + \frac{A_{020}}{6} \right] = 0$$

$$\Downarrow$$

$$A_{020}^{opt} = 6 \left[ \langle W_s \rangle - 2 \langle W_s R_\rho^2 \rangle \right] = - \frac{u^2 \Delta z'_{opt}}{2}$$

$$\Delta z'_{opt} = \frac{12}{u^2} \left[ 2 \langle W_s R_\rho^2 \rangle - \langle W_s \rangle \right]$$

$$D^{opt} = 1 - k^2 \left[ \Delta W_s^2 + A_{020}^{opt} \left( - \frac{A_{020}^{opt}}{6} \right) + \frac{A_{020}^{opt\,2}}{12} \right]$$

$$= 1 - k^2 \left[ \Delta W_s^2 - \frac{A_{020}^{opt\,2}}{12} \right]$$

sfér. vada

$$W_s = A_{040} R_p^4$$

$$\langle W_s \rangle = A_{040} \langle R_p^4 \rangle = \frac{A_{040}}{3}$$

$$\langle W_s R_p^2 \rangle = A_{040} \langle R_p^6 \rangle = A_{040} \frac{1}{\pi} 2\pi \frac{R_p^8}{8} \Big|_0^1 = \frac{A_{040}}{4}$$

$$\langle W_s^2 \rangle = A_{040}^2 \langle R_p^8 \rangle = A_{040}^2 \frac{1}{\pi} 2\pi \frac{R_p^{10}}{10} \Big|_0^1 = \frac{A_{040}^2}{5}$$

$$\Delta W_s^2 = A_{040}^2 \left( \frac{1}{5} - \frac{1}{9} \right) = \frac{4}{45} A_{040}^2$$

opt. zaostření

$$A_{020}^{opt} = 6 \left[ \frac{A_{040}}{3} - \frac{A_{040}}{2} \right] = -A_{040}$$

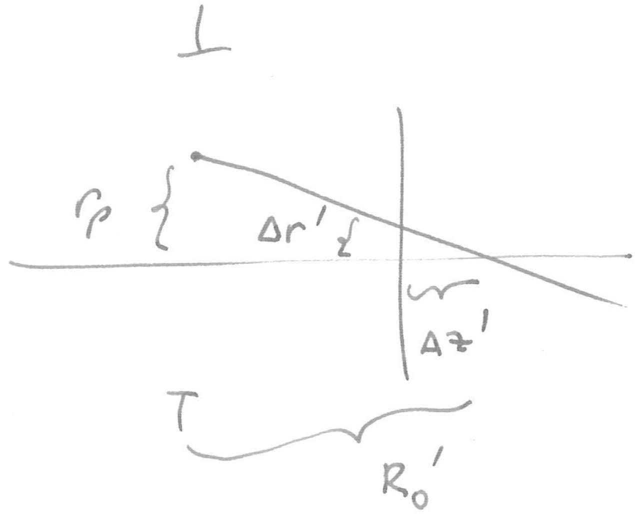
$$\Delta z'_{opt} = \frac{12}{u^2} \frac{A_{040}}{6} = \frac{2}{u^2} A_{040}$$

$$D = 1 - k^2 \Delta W_s^2 = 1 - \frac{4}{45} k^2 A_{040}^2$$

$$D^{opt} = 1 - k^2 A_{040}^2 \underbrace{\left( \frac{4}{45} - \frac{1}{12} \right)}_{\frac{1}{180}}$$

$$1 - D^{opt} = \frac{1-D}{16}$$

podélná složka vad.




$$\frac{\Delta z'}{R_0'} = \frac{\Delta r'}{r_p}$$

$$\Delta z' = \frac{R_0'}{r_p}\Delta r' = \frac{R_0'}{\rho_p R_p}\Delta r' = \frac{1}{u R_p}\Delta r'$$

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$$n' u \Delta x' = A_{040}\frac{\partial R_p^4}{\partial X_p} = 4 A_{040} R_p^3 \frac{X_p}{R_p}$$

$$n' u \Delta y' = A_{040}\frac{\partial R_p^4}{\partial Y_p} = 4 A_{040} R_p^3 \frac{Y_p}{R_p}$$

$$\Delta r' = \sqrt{\Delta x'^2 + \Delta y'^2} = \frac{4 A_{040}}{n' u} R_p^3$$

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$$\Delta z'_s = \frac{1}{u R_p}\frac{4 A_{040}}{n' u} R_p^3 = \frac{4}{n' u^2} A_{040} R_p^2$$

největší přípustné $\Delta z_s$

$$1 - \frac{4}{45} k^2 A_{040}^2 = 0{,}8$$

$$\frac{4}{45} k^2 A_{040}^2 = \frac{2}{10}$$

$$k^2 A_{040}^2 = \frac{90}{40} = \frac{9}{4}$$

$$\frac{4\pi^2}{\lambda^2} A_{040}^2 = \frac{9}{4}$$

$$A_{040} = \sqrt{\frac{9\lambda^2}{16\pi^2}} = \frac{3}{4}\frac{\lambda}{\pi}$$

$$\Delta z_s^{max}\Big|_{R_p=1} = \frac{4}{n' u^2} A_{040} = \frac{4}{n' u^2}\,\frac{3}{4}\,\frac{\lambda}{\pi} = \frac{\lambda}{n' u^2}\left(\frac{3}{\pi}\right) \sim 1$$

$$\Delta z_s^{max} \simeq \pm \frac{\lambda}{n' u^2}$$