interferenční zákon

(28)

$$E_p = E_1 + E_2$$

$$I = \langle E_p E_p^* \rangle = \langle E_1 E_1^* \rangle + \langle E_2 E_2^* \rangle + \langle E_1 E_2^* \rangle + c.c$$

$$= I_1 + I_2 + 2 Re\{\underbrace{\langle E_1 E_2^* \rangle}_{\Gamma_{12}}\} \ominus$$

$$\gamma_{12} = \frac{\Gamma_{12}}{\sqrt{I_1 I_2}}, \quad \gamma_{12} = |\gamma_{12}| e^{i\phi_{12}}$$

$$\ominus \quad I_1 + I_2 + 2\sqrt{I_1 I_2}\, |\gamma_{12}| \cos\phi_{12}$$

koh.

$$I = |E_1 + E_2|^2 = I_1 + I_2 + \sqrt{I_1 I_2}\, e^{i(\phi_1 - \phi_2)} + \sqrt{I_1 I_2}\, e^{-i(\phi_1 - \phi_2)}$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\phi_1 - \phi_2)$$

$$\Rightarrow |\gamma_{12}| = 1$$

nekoh.

$$I = I_1 + I_2 \Rightarrow |\gamma_{12}| = 0$$

van Cittert-Zernike

$$a(x_1) = \int a(\xi)\, \underbrace{e^{-i\frac{k}{2z}(x_1-\xi)^2}}_{h(x_1-\xi)}\, d\xi$$

$$\Gamma(x_1, x_2) = \langle a(x_1)\, a^*(x_2) \rangle$$

$$= \iint h(x_1-\xi)\, h^*(x_2-\xi')\, \underbrace{\langle a(\xi)\, a^*(\xi') \rangle}_{I(\xi)\,\delta(\xi-\xi')}\, d\xi\, d\xi'$$

$$= \int I(\xi)\, \underbrace{h(x_1-\xi)\, h^*(x_2-\xi)}_{e^{-i\frac{k}{2z}(x_1-\xi)^2}\; e^{i\frac{k}{2z}(x_2-\xi)^2}}\, d\xi \;\circeq$$

$$\swarrow \qquad\qquad \searrow$$

$$e^{i\frac{k}{2z}(x_2^2-x_1^2)} \qquad\qquad e^{i\frac{k}{2z}2(x_1\xi - x_2\xi)}$$

$$\circeq \quad e^{i\frac{\pi}{\lambda z}(x_2^2-x_1^2)} \int I(\xi)\, e^{i\underbrace{\frac{2\pi}{\lambda z}(x_1-x_2)}_{p}\xi}\, d\xi$$

kondenzor

$$a(x_1) = \int a_z(\varsigma)\, h_k(x_1-\varsigma)\, d\varsigma$$

zdroj — kondenzor

$$h_k = \mathcal{F}\{P(X_p)\}$$

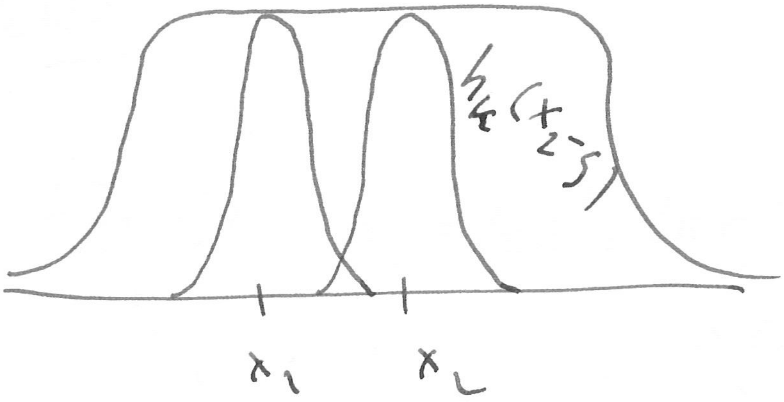
normální souřadnice:

$$h_k(x_1) = \int P(x_p)\, e^{+i\frac{2\pi}{\lambda z_p} x_p x_1}\, dx_p$$

$$\Gamma(P_1, P_2) = \langle a(x_1)\, a^*(x_2) \rangle$$

$$= \iint \underbrace{\langle a_z(\varsigma)\, a_z^*(\varsigma') \rangle}_{I_z(\varsigma)\,\delta(\varsigma-\varsigma')} h_k(x_1-\varsigma)\, h_k^*(x_2-\varsigma')\, d\varsigma\, d\varsigma'$$

$$= \int I_z(\varsigma)\, h_k(x_1-\varsigma)\, h_k^*(x_2-\varsigma)\, d\varsigma \quad =$$



$$\propto \int h_\varepsilon(x_1-\xi)\, h_\varepsilon^*(x_2-\xi)\, d\xi$$

$$= \iiint P(x_p) P^*(x_p')\, e^{i\frac{2\pi}{\lambda z_p} x_p (x_1-\xi)}\, e^{-i\frac{2\pi}{\lambda z_p} x_p'(x_2-\xi)}\, d\xi\, dx_p\, dx_p'$$

$$= \iint P(x_p) P^*(x_p')\, e^{i\frac{2\pi}{\lambda z_p}(x_p x_1 - x_p' x_2)} \underbrace{\int e^{i\frac{2\pi}{\lambda z_p}(x_p - x_p')}\, d\xi}_{\delta\left(\frac{x_p-x_p'}{\lambda z_p}\right) \sim \delta(x_p - x_p')}\, dx_p\, dx_p'$$

$$\Gamma_{12} = \iint |P(x_p)|^2\, e^{i\frac{2\pi}{\lambda z_p} x_p (x_1 - x_2)}\, dx_p$$

tj. totéž, jako Γ_{12} od nekoh. zdroje

v pupile !

objektiv

objektiv

$$a^{(j)}(P') = \frac{2 J_1(\nu_j)}{\nu_j}, \quad \nu_j = 2\pi R' = \frac{2\pi r_j' \rho_p}{\lambda z_p}$$

$\sin \alpha_0$

$$r_j' = \sqrt{(x'-x_j')^2 + (y'-y_j')^2}$$

$$I^{(j)}(P') = \left[\frac{2 J_1(\nu_j)}{\nu_j}\right]^2$$

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stupeň koherence $\gamma(P_1, P_2)$

$$r_j' \to r_{12} = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

$$\alpha_0 \to \alpha_k, \quad \sin\alpha_k = \frac{\rho_p}{z_p} \text{ - kondensor}$$

$$\gamma(P_1, P_2) = \frac{2 J_1(u_{12})}{u_{12}}, \quad u_{12} = \frac{2\pi}{\lambda} r_{12} \sin\alpha_k$$

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intenzita obrazu (symetrická pupila konden.)

$$I(P') = I^{(1)} + I^{(2)} + 2\sqrt{I^{(1)} I^{(2)}}\, |\gamma_{12}|$$

$$= \left[\frac{2 J_1(\nu_1)}{\nu_1}\right]^2 + \left[\frac{2 J_1(\nu_2)}{\nu_2}\right]^2 + 2\, \frac{2 J_1(\nu_1)}{\nu_1} \frac{2 J_1(\nu_2)}{\nu_2} \frac{2 J_1(u_{12})}{u_{12}}$$

nebo

$$u_{12} = v_{12} \left(\frac{\sin \alpha_k}{\sin \alpha_0} \right) m$$

$$v_{12} = \frac{2\pi}{\lambda} r_{12} \sin \alpha_0$$ (nezávisí na apertuře kondenzoru)

$$I(P') = I^{(1)} + I^{(2)} + 2\sqrt{I^{(1)} I^{(2)}} \; \frac{2 J_1(m v_{12})}{m v_{12}}$$

spec.

(a) malá apert. kond. $m \to 0$ (koher. osvětl.)

$$\frac{2 J_1(m v_{12})}{m v_{12}} \to 1 \, .$$

$$I(P') = \left[\sqrt{I^{(1)}} + \sqrt{I^{(2)}} \right]^2$$

(b) nekoh. zobrazení $\gamma_{12} = 0$

$J_1(u_{12}) = 0 \Rightarrow r_{12} = d(P_1, P_2) =$ poloměr difr. min. kond.

$$\left\{ \begin{array}{l} m = 1 \Rightarrow \text{pol. difr. min. kond.} = \text{p.d.m. objektivu} \\ \text{zvětš. obj} = 1 \Rightarrow d(P_1', P_2') = d(P_1, P_2) \end{array} \right.$$

$\Rightarrow$ $d(P_1', P_2') =$ p.d.m. objektivu