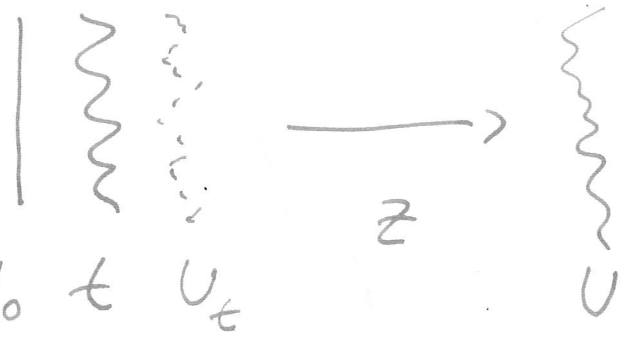
I

Talbotův jev – koherentní

L9



amplitudová mřížka

$$U_t(\xi) = t(\xi)\, U_0(\xi) \propto t(\xi)$$ (rovinná vlna)

$$t(\xi) = \frac{1}{2}\left[1 + m \cos(2\pi \xi / \Lambda)\right]$$

$$U(x) = \int U_t(\xi)\, e^{-i \frac{k}{2z}(x-\xi)^2}\, d\xi$$

$$= \int U_t(\xi)\, h(x-\xi)\, d\xi$$

$$h(x) = e^{-i \frac{k}{2z} x^2}$$

$$U(x) = t(x) * h(x)$$

$$\mathcal{U}(\nu_x) = T(\nu_x)\, H(\nu_x)$$

$$T(\nu_x) = \int t(x)\, e^{i 2\pi \nu_x x}\, dx$$

$$H(\nu_x) = \int h(x)\, e^{i 2\pi \nu_x x}\, dx \propto e^{i\pi\lambda z \nu_x^2}$$

$$t(\xi) = \frac{1}{2} + \frac{m}{4} e^{i\frac{2\pi\xi}{L}} + \frac{m}{4} e^{-i\frac{2\pi\xi}{L}}$$

$$\downarrow \text{FT}$$

$$T(\nu_x) = \frac{1}{2}\delta(\nu_x) + \frac{m}{4}\delta\left(\nu_x + \frac{1}{L}\right) + \frac{m}{4}\delta\left(\nu_x - \frac{1}{L}\right)$$

$$U(x) = \mathcal{F}^{-1}\left\{H(\nu_x)\, T(\nu_x)\right\}$$

$$= \frac{1}{2}\int e^{i\pi\lambda z \nu_x^2}\, \delta(\nu_x)\, e^{-i 2\pi \nu_x x}\, d\nu_x$$

$$+ \frac{m}{4}\int e^{i\pi\lambda z \nu_x^2}\, \delta\left(\nu_x + \frac{1}{L}\right) e^{-i 2\pi \nu_x x}\, d\nu_x + \dots$$

$$= \frac{1}{2} + \frac{m}{4} e^{i\pi\frac{\lambda z}{L^2}} e^{i 2\pi \frac{x}{L}} + \frac{m}{4} e^{i\pi\frac{\lambda z}{L^2}} e^{-i 2\pi\frac{x}{L}}$$

$$= \frac{1}{2}\left[1 + m\cos\left(2\pi\frac{x}{L}\right) e^{i\frac{\pi\lambda z}{L^2}}\right]$$

① T. obraz

$$e^{i\frac{\pi\lambda z}{L^2}} = 1 \Rightarrow \frac{\pi\lambda z}{L^2} = 2n\pi, \quad n = 0, 1, \dots$$

$$z = \frac{2nL^2}{\lambda}$$

② T. reverzní obraz

$$e^{i\frac{\pi\lambda z}{L^2}} = -1 \Rightarrow \frac{\pi\lambda z}{L^2} = (2n+1)\pi, \quad n = 0, 1, \dots$$

③ T. subobraz

$$e^{i\frac{\pi\lambda z}{L^2}} = \pm i \Rightarrow \frac{\pi\lambda z}{L^2} = (2n+1)\frac{\pi}{2}, \quad n = 0, 1, \dots$$

$$U(x) = \frac{1}{2}\left[1 \pm i m \cos\left(\frac{2\pi x}{L}\right)\right]$$

$$I(x) = \frac{1}{4}\left[1 + m^2\cos^2\left(\frac{2\pi x}{L}\right)\right] \;(=)$$

$$\cos^2\varphi = \frac{\cos 2\varphi + 1}{2}$$

$$(=)\; \frac{1}{4}\left[\underbrace{1 + \frac{m^2}{2} + \frac{m^2}{2}}_{\text{snížený kontrast}}\cos\frac{4\pi x}{L}\right)$$

obecný periodický předmět

$$t(\xi+L) = t(\xi)$$

$$t(\xi) = \mathcal{F}^{-1}\{T(\nu_x)\} = \int T(\nu_x)\, e^{-i2\pi\xi\nu_x}\, d\nu_x$$

$$t(\xi+L) = \int T(\nu_x)\, e^{-i2\pi\xi\nu_x}\, e^{-i2\pi L\nu_x}\, d\nu_x$$

$$\Rightarrow \quad e^{-i2\pi L\nu_x} = 1 \quad \forall\, \nu_x : T(\nu_x) \neq 0$$

podmínka na frekvence

$$2\pi L\nu_x = 2\pi m \,, \quad m = 0, 1, \dots$$

$$\nu_x = \frac{m}{L} \qquad \text{diskrétní spektrum}$$

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samozobrazení

$$H(\nu_x) = 1 \quad \forall\ \nu_x = \frac{m}{L} \,, \quad m = 0, 1, \dots$$

$$H\left(\frac{m}{L}\right) = e^{i\pi\lambda z\left(\frac{m}{L}\right)^2} = 1$$

$$\frac{\lambda z}{L^2}m^2 = 2k \,, \ k = 0, 1, \dots \quad \Rightarrow \quad \frac{\lambda z}{2L^2} = n \,, \ n = 0, 1, \dots$$

$$\boxed{z = \frac{2nL^2}{\lambda}}$$

Talbot - část. koherence

$$U_t(\xi) = t(\xi)\, U_0(\xi)$$

$$U(x) = \int U_t(\xi)\, h(x-\xi)\, d\xi$$

$$I(x) = \iint h(x-\xi)\, h^*(x-\xi')\, U_t(\xi)\, U_t^*(\xi')\, d\xi\, d\xi'$$

část. koh. osvětlení

$$I(x) = \iint h(x-\xi)\, h^*(x-\xi')\, t(\xi)\, t^*(\xi')\, \underbrace{\langle U_0(\xi)\, U_0^*(\xi')\rangle}_{\Gamma(\xi,\xi')}\, d\xi\, d\xi'$$

simulace část. koh. zdroje

$$\Gamma(\xi,\xi') \propto e^{-\frac{(\xi-\xi')^2}{2\sigma^2}}$$

$$\sigma \ll 1 \Rightarrow \Gamma(\xi,\xi') \propto \delta(\xi-\xi') \quad \text{nekoh.}$$

$$\sigma \gg 1 \Rightarrow \Gamma(\xi,\xi') \propto 1 \quad \text{koh.}$$