frekvenční analýza

(L10)

podmínky

$$U(x) = \int U(x_0)\,\delta(x-x_0)\,dx_0$$

linearita

$$U'(x') = \int U(x_0)\,G(x', x_0)\,dx_0$$

izoplanaze

$$U'(x') = \int U(x_0)\,G(x'-x_0)\,dx_0$$

$$\downarrow \mathcal{F}$$

$$\bar{U}'(\nu_x) = \bar{U}(\nu_x)\,\bar{G}(\nu_x)$$

izoplanaze

$$U'(x') = \int U(x_0)\,G(x'-x_0)\,dx_0$$

$$U(x_0) \to U(x_0-a)$$

$$U''(x') = \int U(x_0-a)\,G(x'-x_0)\,dx_0$$

$$= \int U(x_0')\,G(x'-a-x_0')\,dx_0'$$

$$= U'(x'-a)$$

rovinné vlny

$$e^{i2\pi(\nu_x x + \nu_y y)} \longleftrightarrow e^{i(k_x x + k_y y + k_z z)}$$

$$2\pi \nu_x = k_x \quad , \quad k_x = \frac{2\pi}{\lambda}\cos\alpha$$

$$2\pi \nu_y = k_y \quad , \quad k_y = \frac{2\pi}{\lambda}\cos\beta$$

$$\nu_x = \frac{\cos\alpha}{\lambda} \quad , \quad \nu_y = \frac{\cos\beta}{\lambda}$$

$$k_z = \sqrt{k^2 - k_x^2 - k_y^2} = k\sqrt{1 - \left(\frac{k_x}{k}\right)^2 - \left(\frac{k_y}{k}\right)^2}$$

$$= k\sqrt{1 - \cos^2\alpha - \cos^2\beta}$$

$$\cos^2\alpha + \cos^2\beta < 1$$

$$\lambda^2\nu_x^2 + \lambda^2\nu_y^2 < 1$$

$$\sqrt{\nu_x^2 + \nu_y^2} < \frac{1}{\lambda}$$ max. přenášená frekvence

zobrazení

$$U'(x') = \int U(x_0)\, G(x' - m x_0)\, dx_0$$

geom. zobr. $G(x' - m x_0) \to \delta(x' - m x_0)$

$$U'(x') = \int U(x_0)\, \delta(x' - m x_0)\, dx_0$$

$$\propto \int U\left(\frac{x_0'}{m}\right) \delta(x' - x_0')\, dx_0'$$

$$= U\left(\frac{x'}{m}\right)$$ zvětšený obraz

frekvenční analýza

$$U'(x') \propto \int U\left(\frac{x_0'}{m}\right) G(x' - x_0')\, dx_0'$$

$$\bar{U}'(\nu_x) = \mathcal{F}\{U'(x')\}$$

$$= \int U\left(\frac{x_0'}{m}\right) \underbrace{\int G(x' - x_0')\, e^{i 2\pi x' \nu_x}\, dx'}\, dx_0' \;=$$

$$\int G(x)\, e^{i 2\pi x \nu_x}\, e^{i 2\pi x_0' \nu_x}\, dx'$$ $x' - x_0' = x$

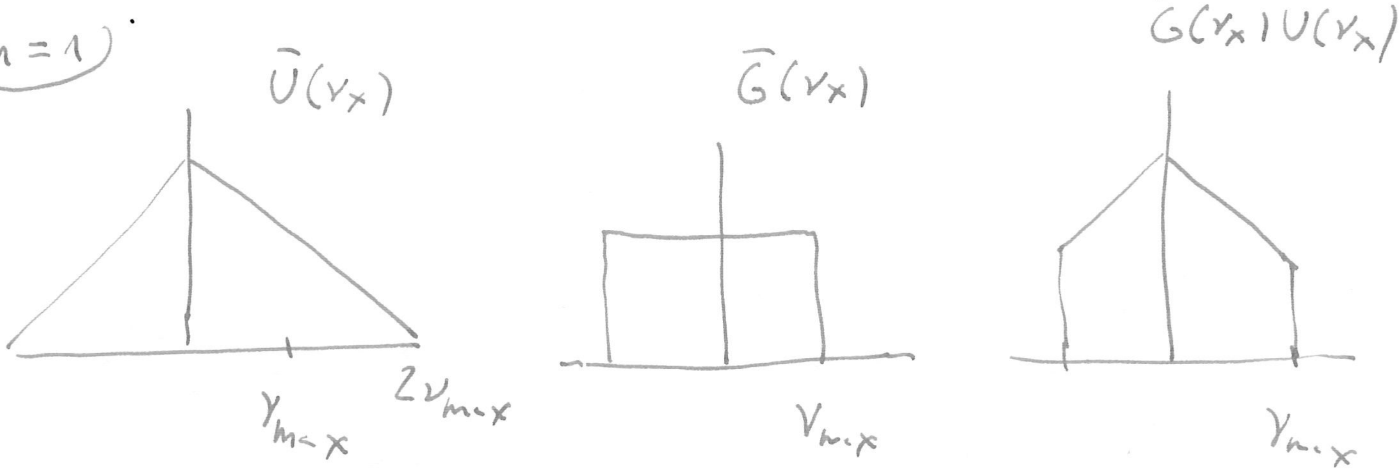
$$= e^{i 2\pi x_0' \nu_x}\, \bar{G}(\nu_x)$$

$$= \bar{G}(\nu_x) \int U\Big(\underbrace{\tfrac{x_0'}{m}}_{x_0}\Big)\, e^{i 2\pi x_0' \nu_x}\, dx_0'$$

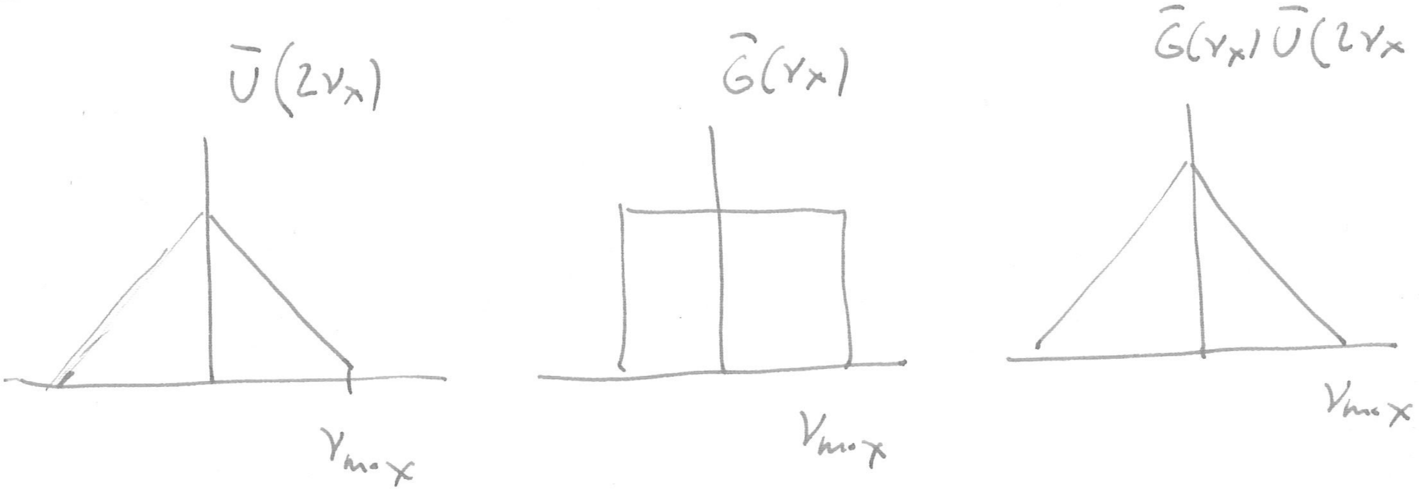
$$= \bar{G}(\nu_x) \underbrace{\int U(x_0)\, e^{i 2\pi \nu_x m x_0}\, dx_0}_{\bar{U}(m\nu_x)}$$

$$\bar{U}'(\nu_x) = \bar{G}(\nu_x)\, \bar{U}(m\nu_x)$$

(m = 1)



(m = 2)



(5a+6a

alternativně koh. + nekoh.

$$G(X') = \int P(X_p)\, e^{i2\pi X_p X'} dX_p = \mathcal{F}_{X_p}\{P(X_p)\}$$

$$\bar{G}(\bar{\nu}_x) = \int G(X')\, e^{i2\pi \bar{\nu}_x X'} dX' = \mathcal{F}_{X'}\{G(X')\}$$

$$\bar{G}(\nu_x) = \mathcal{F}_{x'}\{G(x')\}$$

$$\left.\begin{array}{l} \bar{\nu}_x \text{ konjug. k } X' \\ \nu_x \text{ konjug. k } x' \end{array}\right\} \quad X' = x' \frac{\rho_p}{\lambda p} \Rightarrow \bar{\nu}_x = \nu_x \frac{\lambda p}{\rho_p}$$

$$\bar{G}(\bar{\nu}_x) = P(-\bar{\nu}_x) \Rightarrow \bar{G}(\nu_x) = P\left(-\frac{\lambda p}{\rho_p}\nu_x\right)$$

$$\bar{T}(\bar{\nu}_x) = \mathcal{F}_{X'}\{T(X')\} = \mathcal{F}_{X'}\{G(X')\, G^*(X')\}$$

$$= \mathcal{F}_{X'}\{G(X')\} * \mathcal{F}_{X'}\{G(X')\}$$

$$= P(-\bar{\nu}_x) * P(-\bar{\nu}_x)$$

$$= \int P(-X_p)\, P^*(-X_p + \bar{\nu}_x)\, dX_p$$

$$= \int P(X_p)\, P^*(X_p + \bar{\nu}_x)\, dX_p$$

OFP koherentní

$$G(X') = \mathcal{F}\{P(X_p)\} = \int P(X_p)\, e^{i 2\pi X_p X'} dX_p$$

$$G(x') = \int P(X_p)\, e^{i \frac{2\pi \rho_p}{\lambda p} X_p x'} dX_p$$

$$\propto \int P\left(\frac{x_p}{\rho_p}\right) e^{i \frac{2\pi}{\lambda p} x_p x'} dx_p$$

$$\propto \int P\left(\frac{\lambda p}{\rho_p} x_p'\right) e^{i 2\pi x_p' x'} dx_p'$$

$$\bar{G}(\nu_x) = \mathcal{F}\{G(x')\}$$

$$= \int P\left(\frac{\lambda p}{\rho_p} x_p'\right) \underbrace{\int e^{i 2\pi x_p' x'} e^{i 2\pi \nu_x x'} dx'}_{\delta(x_p' + \nu_x)} dx_p'$$

$$= P\left(-\frac{\lambda p}{\rho_p} \nu_x\right)$$

kruhová pupila

$$\left(\frac{\lambda p}{\rho_p} \nu_x\right)^2 + \left(\frac{\lambda p}{\rho_p} \nu_y\right)^2 \leq 1 \quad \Rightarrow \quad \nu_x^2 + \nu_y^2 \leq \frac{\rho_p^2}{\lambda^2 p^2}$$

$$\nu_{max} = \sqrt{\nu_x^2 + \nu_y^2}\Big|_{max} = \frac{\rho_p}{\lambda p} = \frac{\sin\alpha}{\lambda} = \frac{NA}{\lambda}$$

OFP nekoherentní

$$\bar{T}(\nu_x) = \int T(x') e^{i 2\pi \nu_x x'} dx'$$

$$\propto \int \underbrace{T\left(\frac{p\lambda}{\rho_p} X'\right)}_{\equiv T(X')} e^{i 2\pi \overbrace{\left(\nu_x \frac{p\lambda}{\rho_p}\right)}^{\bar{\nu}_x} X'} dX'$$

$$\bar{T}(\bar{\nu}_x) = \int T(X') e^{i 2\pi \bar{\nu}_x X'} dX'$$

$$T(X') = G(X') G^*(X')$$

$$\bar{T}(\bar{\nu}_x) = \bar{G}(\bar{\nu}_x) * \bar{G}(\bar{\nu}_x) \qquad \text{křiž. korelace}$$

$$\bar{G}(\bar{\nu}_x) = \int G(X') e^{i 2\pi \bar{\nu}_x X'} dX' = P(-\bar{\nu}_x)$$

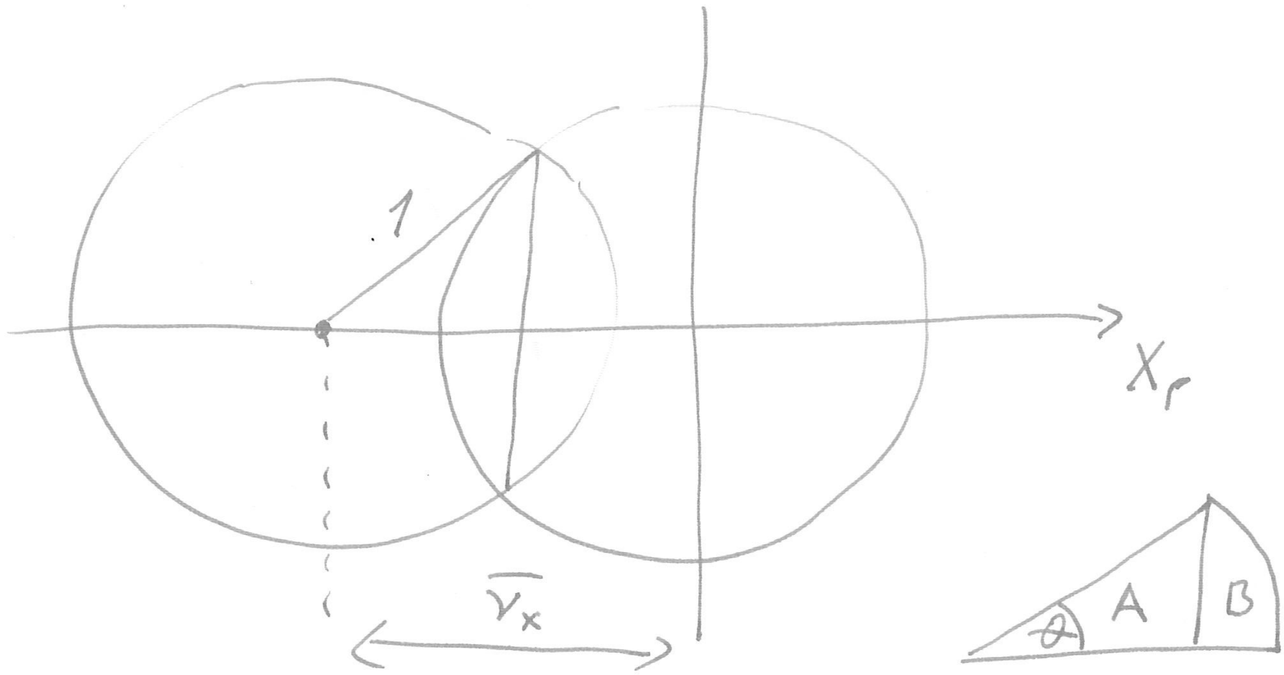
$$\bar{T}(\bar{\nu}_x) = P(-\bar{\nu}_x) * P(-\bar{\nu}_x)$$

$$= \int P(-X_p) P^*[-(X_p - \bar{\nu}_x)] dX_p$$

$$= \int P(-X_p) P^*(-X_p + \bar{\nu}_x) dX_p$$

$$= \int P(X_p) P^*(X_p + \bar{\nu}_x) dX_p$$

OFP – kruhová pupila



$$S_{A+B} = \frac{\theta}{2\pi}\pi \quad , \quad \theta = \arccos \frac{\bar{\nu}_x}{2}$$

$$S_A = \frac{\frac{\bar{\nu}_x}{2} \cdot \sqrt{1-\left(\frac{\bar{\nu}_x}{2}\right)^2}}{2}$$

$$S_B = S_{A+B} - S_A = \frac{\arccos \frac{\bar{\nu}_x}{2}}{2} - \frac{\frac{\bar{\nu}_x}{2}\sqrt{1-\left(\frac{\bar{\nu}_x}{2}\right)^2}}{2}$$

$$\bar{T}_N(\bar{\nu}_x) = \frac{4 S_B}{S_P} = \frac{2}{\pi}\left[\arccos \frac{\bar{\nu}_x}{2} - \frac{\bar{\nu}_x}{2}\sqrt{1-\left(\frac{\bar{\nu}_x}{2}\right)^2}\right]$$