Wienerův filtr

$$i(x) = s(x) * o(x) + n(x) \Rightarrow I(f) = S(f)O(f) + N(f)$$

filtr: $\hat{o}(x) = h(x) * i(x) \Rightarrow \hat{O}(f) = H(f)I(f)$

minimalizuji: $\epsilon(f) = \langle |O(f) - \hat{O}(f)|^2 \rangle$

$$\epsilon(f) = \langle |O(f) - H(f)I(f)|^2 \rangle$$

$$= \langle |O(f) - H(f)[S(f)O(f) + N(f)]|^2 \rangle$$

$$= \langle |O(f)[1 - H(f)S(f)] - H(f)N(f)|^2 \rangle$$

$$= [1 - H(f)S(f)][1 - H(f)S(f)]^* \underbrace{\langle |O(f)|^2 \rangle}_{\phi_o}$$

$$+ H(f)H(f)^* \underbrace{\langle |N(f)|^2 \rangle}_{\phi_n}$$

$$\frac{d\epsilon}{dH^*} = -[1 - H(f)S(f)]\, S^*(f)\, \phi_o + H(f)\phi_n = 0$$

$$H(f)[|S(f)|^2 \phi_o + \phi_n] = S^*(f)\phi_o$$

$$\boxed{H(f) = \frac{S^*(f)}{|S(f)|^2 + \frac{\phi_n}{\phi_o}}}$$

chirp

autokorelace

$$s(x) = e^{-i\pi\alpha x^2} \otimes e^{-i\pi\alpha x^2}$$

$$s(x) = \int e^{-i\pi\alpha\xi^2} e^{i\pi\alpha(\xi - x)^2} d\xi$$

$$= e^{i\pi\alpha x^2} \underbrace{\int e^{-i2\pi\alpha\xi x} d\xi}_{\delta(\alpha x)}$$

$$\propto \delta(x)$$

jinak

konst. fáze

$$\mathcal{F}\{ e^{-i\pi\alpha x^2} \} \propto \frac{1}{\sqrt{\alpha}} e^{i\pi \frac{f_x^2}{\alpha}}$$

$$\mathcal{F}\{ s(x) \} = \frac{1}{\alpha}$$

$$s(x) = \frac{\delta(x)}{\alpha}$$

SAR chirp

$$s(x) = e^{-i\pi\alpha x^2}\,\mathrm{rect}\left(\frac{x}{D_{eff}}\right) \circledast e^{-i\pi\alpha x^2}, \quad \alpha = \frac{2}{\lambda_r r_0}$$

$$s(x) \equiv \int e^{-i\pi\alpha\varsigma^2}\,\mathrm{rect}\left(\frac{\varsigma}{D_{eff}}\right) e^{i\pi\alpha(\varsigma - x)^2}\, d\varsigma$$

$$= e^{i\pi\alpha x^2} \int \cancel{e^{-i\pi\alpha\varsigma^2}}\,\mathrm{rect}\left(\frac{\varsigma}{D_{eff}}\right) \cancel{e^{i\pi\alpha\varsigma^2}}\, e^{-i2\pi \overbrace{\alpha x}^{f_x} \varsigma}\, d\varsigma$$

$$= e^{i\pi\alpha x^2}\, \mathcal{F}\left\{\mathrm{rect}\left(\frac{\varsigma}{D_{eff}}\right)\right\}_{f_x \to \alpha x}$$

$$= e^{i\pi\alpha x^2}\, D_{eff}\, \mathcal{F}\left\{\mathrm{rect}(\varsigma)\right\}_{f_x \to D_{eff}\alpha x}.$$

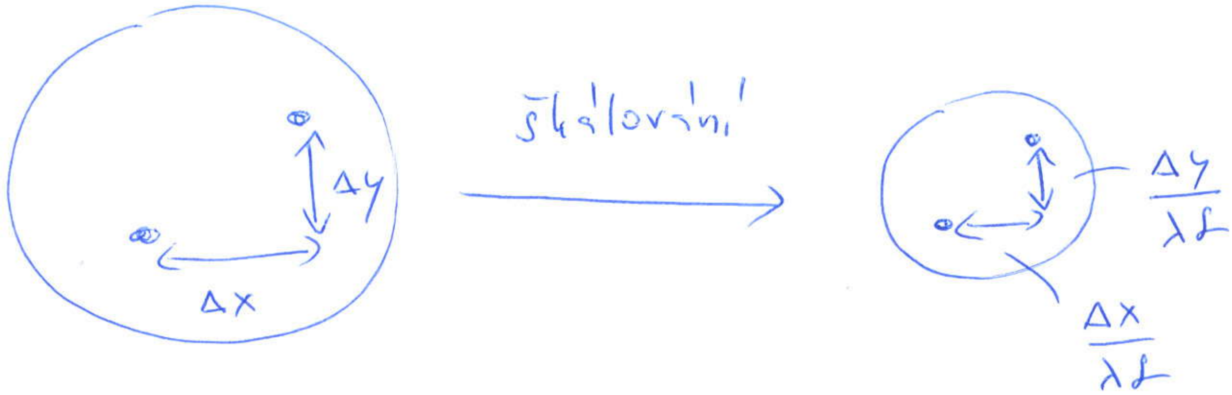
$$= D_{eff}\, e^{i\pi\alpha x^2}\, \mathrm{sinc}(D_{eff}\alpha x)$$

$$|s(x)|^2 = D_{eff}^2\, \mathrm{sinc}(D_{eff}\alpha x)^2$$

$$= D_{eff}^2\, \mathrm{sinc}^2\left(\frac{2D_{eff}}{\lambda_r r_0} x\right)$$

syntéza apertury

$$\mathcal{H}(f_x, f_y) \sim P(\lambda f f_x, \lambda f f_y) \circledast P(\lambda f f_x, \lambda f f_y)$$



1D

$$P(x) \sim \delta\left(x - \frac{\Delta x}{2}\right) + \delta\left(x + \frac{\Delta x}{2}\right)$$

$$\delta(x+\alpha) \circledast \delta(x+\beta) \equiv \int \delta(\xi + \alpha)\, \delta(\xi - x + \beta)\, d\xi$$

$$= \delta(-\alpha - x + \beta) = \delta(x + \alpha - \beta)$$

$$P(x) \circledast P(x) \sim \left[\delta\left(x - \frac{\Delta x}{2}\right) + \delta\left(x + \frac{\Delta x}{2}\right)\right] \circledast \left[\cdots\right]$$

$$\sim \delta(x) + \frac{1}{2}\delta(x - \Delta x) + \frac{1}{2}\delta(x + \Delta x)$$

$$\mathcal{H}(f_x) \sim \delta(\lambda f f_x) + \frac{1}{2}\delta(\lambda f f_x - \Delta x) + \frac{1}{2}\delta(\lambda f f_x + \Delta x)$$

$$\sim \delta(f_x) + \frac{1}{2}\delta\Big(f_x - \underbrace{\frac{\Delta x}{\lambda f}}_{\Delta f_x}\Big) + \frac{1}{2}\delta\left(f_x + \frac{\Delta x}{\lambda f}\right)$$

výsl. intenzita

$$i(x) = \mathcal{F}^{-1}\{\mathcal{H}(f_x)\, O(f_x)\}$$

$$|O(f_x)|\, e^{i\phi(f_x)}$$

normalizace

$$O(f_x = 0) = 1$$

$$\mathcal{F}^{-1}\{\delta(f_x)\, O(f_x)\} = \int \delta(f_x)\, O(f_x)\, e^{i2\pi f_x x}\, df_x = 1$$

$$\mathcal{F}^{-1}\{\delta(f_x - \Delta f_x)\, O(f_x)\} = \int \delta(f_x - \Delta f_x)\, |O(f_x)|\, e^{i2\pi f_x x + i\phi(f_x)}\, df_x$$

$$= |O(\Delta f_x)|\, e^{i[2\pi\, \Delta f_x x + \phi(\Delta f_x)]}$$

$$\mathcal{F}^{-1}\{\delta(f_x + \Delta f_x)\, O(f_x)\} = |O(-\Delta f_x)|\, e^{i[2\pi(-\Delta f_x)x + \phi(-\Delta f_x)]}$$

$$= |O(\Delta f_x)|\, e^{-i[2\pi \Delta f_x x + \phi(\Delta f_x)]}$$

$$i(x) \sim 1 + |O(\Delta f_x)| \cos[2\pi\, \Delta f_x x + \phi(\Delta f_x)]$$