Bayesova věta

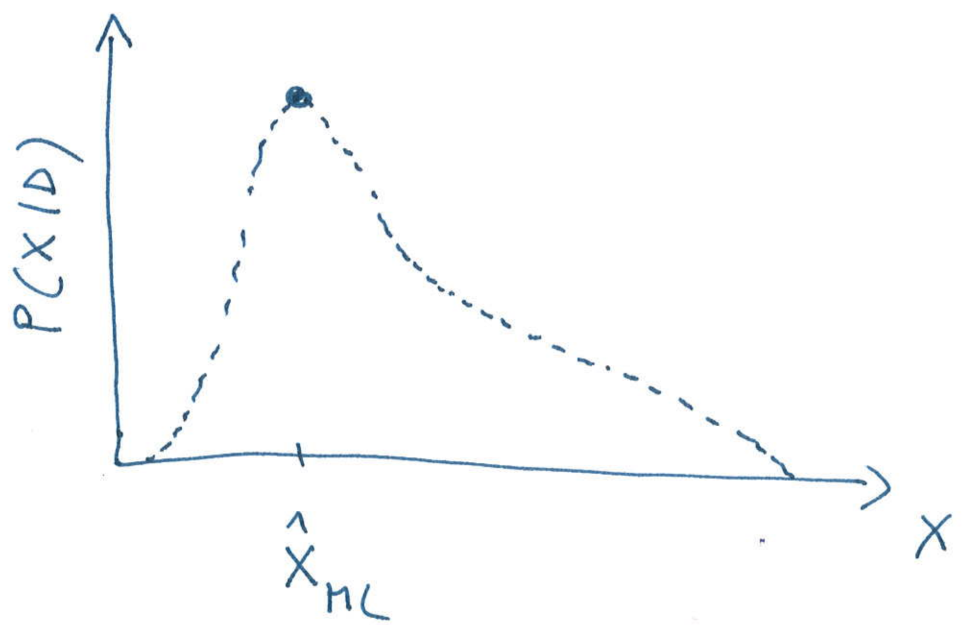
$$P(D \cap X) = P(D|X)\,P(X)$$

$$\parallel$$

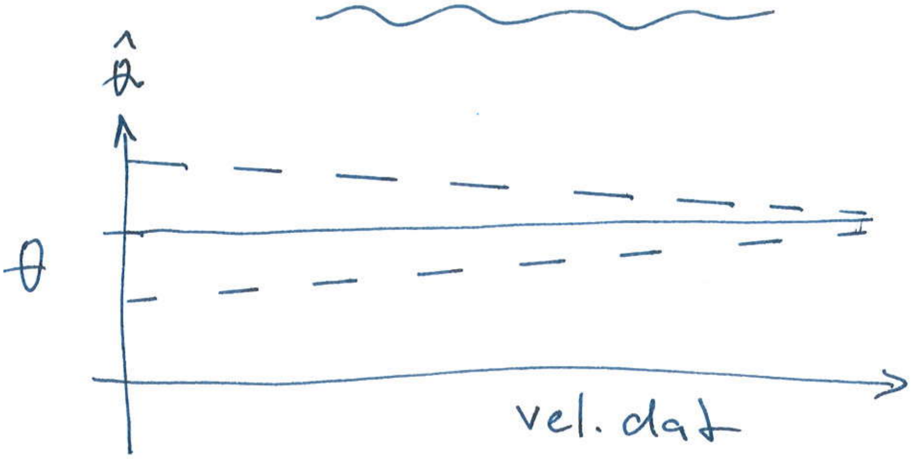
$$P(X \cap D) = P(X|D)\,P(D)$$

$$P(X|D) = \frac{P(D|X)\,P(X)}{P(D)}$$

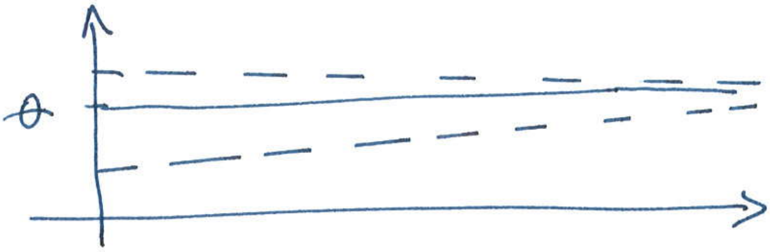
bodový odhad



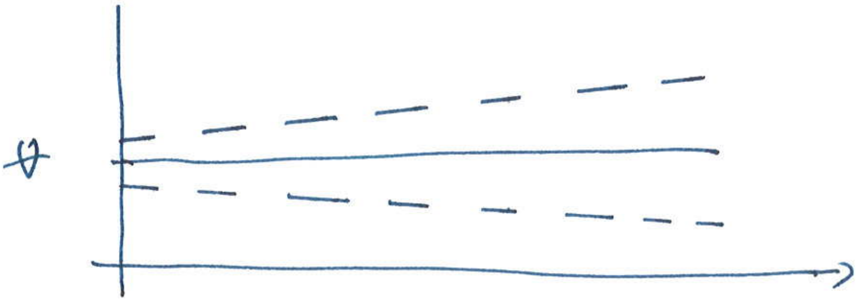
estimátory



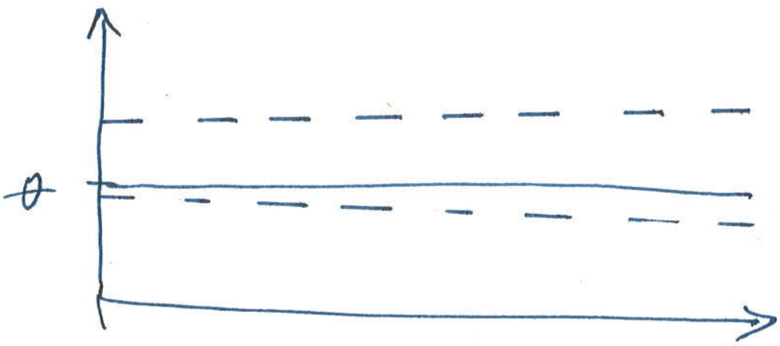
konzistentní
+ nevychýlený



konzistentní
+ vychýlený



nekonz.
+ nevychýlený



nekonz.
+ vychýlený

estimátor variance

stř. hodnota ... d

variance v

$$\hat{d} = \frac{1}{N} \sum_{k=1}^{N} D_k$$

$$\hat{v} = \frac{1}{N} \sum_{k=1}^{N} (D_k - \hat{d})^2$$

$\langle \hat{v} \rangle \neq v$ (bias)

$\hat{d}$ minimalizuje $\sum_k (D_k - x)^2$

proto

$$\frac{1}{N} \sum_k (D_k - \hat{d})^2 < \frac{1}{N} \sum_k (D_k - d)^2$$

a tedy

$$\langle \hat{v} \rangle = \left\langle \frac{1}{N} \sum_k (D_k - \hat{d})^2 \right\rangle < \left\langle \frac{1}{N} \sum_k (D_k - d)^2 \right\rangle = v$$

unbiased : $\frac{1}{N-1} \sum_k (D_k - \hat{d})^2$

pseudoinverze (Moore–Penrose)

$A : M \times N$

pseudoinverze A^-

$$A A^- A = A \quad , \quad A^- A A^- = A^-$$

$$a \in \mathbb{C} : \quad a^- = \frac{1}{a} \; , \; a \neq 0$$

$$a^- = a \; , \; a = 0$$

$$A = U S V^+$$

$$\begin{pmatrix} s_1 & & \\ & s_2 & \\ & & \ddots \end{pmatrix}$$

$$A^- = V S^- U^+$$

$$\begin{pmatrix} s_1^- & & \\ & s_2^- & \\ & & \ddots \end{pmatrix}$$

$$Ax = b$$

$$\hat{x} = A^{-} b$$

- řešení s nejmenší L_2 normou

$$\min \|\hat{x}\|_2$$

- přibližné řešení (OLS)

$$\min \|A\hat{x} - b\|_2$$

EM alg.

$$X_k \geq 0 \quad , \quad \sum_k X_k = 1 \quad , \quad \sum_j D_j = 1$$

$$\langle D_j \rangle = \sum_k \pi_{jk} X_k$$

$$\max \sum_j D_j \log \langle D_j \rangle$$

$$\sum_j D_j \log\left(\sum_k \pi_{jk} Y_k^2 \right) - \lambda \sum_k Y_k^2$$

$$\frac{\partial}{\partial Y_k} = 0 \quad \Rightarrow \quad \sum_j \frac{D_j}{\langle D_j \rangle} \pi_{jk} Y_k = \lambda Y_k \qquad / \cdot Y_k + \text{sum}$$

$$\lambda = 1$$

$$\underbrace{\sum_j \frac{D_j}{\langle D_j \rangle} \pi_{jk}}_{R_k} X_k = X_k$$

extended likelihood $\langle D_j \rangle \to \dfrac{\langle D_j \rangle}{\sum_{j'} \langle D_{j'} \rangle}$

FI – Gauss

$$\mathcal{L} \propto \prod_j e^{-\frac{(n_j - \langle n_j \rangle)^2}{2\sigma_j^2}}$$

$$\log \mathcal{L} \sim -\sum_j \frac{(n_j - \langle n_j \rangle)^2}{2\sigma_j^2}$$

$$\frac{\partial \log \mathcal{L}}{\partial x_k} = -\sum_j \frac{2(n_j - \langle n_j \rangle)}{2\sigma_j^2}\left(-\frac{\partial \langle n_j \rangle}{\partial x_k}\right)$$

$$\left\langle \frac{\partial \log \mathcal{L}}{\partial x_k} \frac{\partial \log \mathcal{L}}{\partial x_\ell} \right\rangle = \sum_j \sum_{j'} \frac{\langle (n_j - \langle n_j \rangle)(n_{j'} - \langle n_{j'} \rangle) \rangle}{\sigma_j^2 \sigma_{j'}^2} \frac{\partial \langle n_j \rangle}{\partial x_k} \frac{\partial \langle n_{j'} \rangle}{\partial x_\ell}$$

$$\langle (n_j - \langle n_j \rangle)(n_{j'} - \langle n_{j'} \rangle) \rangle = \langle n_j n_{j'} \rangle - \langle n_j \rangle \langle n_{j'} \rangle$$

$$= \sigma_j^2 \delta_{jj'}$$

$$F_{k\ell} = \sum_j \frac{1}{\sigma_j^2} \frac{\partial \langle n_j \rangle}{\partial x_k} \frac{\partial \langle n_j \rangle}{\partial x_\ell}$$