Does the Fizeau experiment really test special relativity?

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The motivation and interpretation of the Fizeau experiment are reviewed, and its status as a test of special relativity is discussed. It is shown, with the aid of a simplified, purely mechanical, model of the propagation of light in matter, that the experiment actually cannot discriminate between Galilean and relativistic kinematics.

I. INTRODUCTION

In textbooks on special relativity, it is often asserted that the result of the classical experiment performed by Fizeau (on the propagation of light in a moving liquid)[1,2] is a confirmation of the relativistic law of composition of velocities. This trend goes back to Einstein himself, who states, in a popular book on Relativity (Ref. 3, Chap. 13): "*. . . I shall now draw another conclusion from the theory,* (the theorem of addition of parallel velocities) *. . . which has been most elegantly confirmed by* (the Fizeau) *experiment.*" At the end of the same chapter, the Fizeau experiment is qualified as "*a crucial test in favour of the theory of relativity.*"

On second thought, however, it seems curious that the effect tested by the experiment of Fizeau, Fresnel's "aether-drag law," which, as shall presently be recalled, is an effect of first order in V/c, should be a genuine relativistic effect. All other measurable relativistic effects, whether mechanical effects, or optical effects in vacuum, are of the order of V^2/c^2, and it is not clear why optical effects in continuous media should constitute an exception. Indeed I shall show, with the aid of a simplified model, that *both* the Galilean and the relativistic *microscopic* theories of the propagation of light in a transparent medium yield Fresnel's formula in the first order in V/c, the difference between the two theories being of the order of V^2/c^2.

In Sec. II of this paper, the classical motivation for the experiment of Fizeau and related experiments is reviewed and the experiment itself is briefly described. The status of this experiment as a test of special relativistic kinematics is discussed in Sec. III. Section IV is devoted to the microscopic (Galilean and relativistic) derivations of Fresnel's formula.

Besides its interest in clarifying a historical point, the discussion which follows is, I believe, pedagogically valuable, going to the roots of the principle of relativity, and involving the even more fundamental principle of the unity of science.

II. FRESNEL AETHER-DRAG LAW AND THE FIZEAU EXPERIMENT[4,5]

The first experiments on the propagation of light in moving refractive media are due to Arago (1810), who studied the refraction of light coming from a star. Arago found that the laws of refraction do not depend on the velocity of the refractive medium relative to the source.

The same year, Fresnel worked out a formula to explain the negative result of Arago's experiment. This formula[6] states that the longitudinal velocity v of light traveling in a transparent medium of index n moving with a velocity V is

$$v = c/n + (1 - 1/n^2)V, \qquad (2.1)$$

with c being the velocity of light in vacuum. The velocities c, V, and v in this formula are velocities relative to the Newtonian "absolute" space, which may be approximated by the Copernicus reference frame, defined by fixed stars. However, it is easily seen, using the Galilean law of addition of velocities, that Fresnel's formula remains valid, to first order in w/c, in any Galilean reference frame of velocity w relative to the Copernicus frame.

Arago's experiments were followed by a series of other experiments on the optical properties of refractive media at rest in the laboratory frame. Airy (1871) studied the dependence of the aberration of light in a refractive medium on the index n, with a negative result (no dependence). Hoek (1868) and Mascart (1874) tried to detect the absolute motion of the earth by realizing the interference of two light beams following opposite directions along the same optical circuit, containing a refractive medium fixed in the laboratory (this is a first-order experiment, in contrast with the Michelson experiment of 1881 which is a second-order experiment). The negative results of these experiments were predicted by Fresnel's law, which was thus indirectly corroborated.

However, the most convincing test of Fresnel's law is the direct measure of the difference

$$\delta v = v - c/n = (1 - 1/n^2)V, \qquad (2.2)$$

which was first made by Fizeau in 1851. For a current of water ($n = 4/3$) flowing with a velocity V of the order of 5 m/sec, the relative effect $\delta v/v \simeq 10^{-8}$ can easily be measured by interferometry.

Fizeau realized the interference of two light beams following exactly the same optical circuit through a current of water, but in opposite directions, one beam going with the current and the other against the current. In a slightly modified version of Fizeau's apparatus (Fig. 1), the beam emitted by a source A is divided in two by a semitransparent mirror B; the transmitted beam follows the circuit $BCDEB$, and is then transmitted through B towards F, while the reflected beam follows the opposite circuit $BEDCB$, and is then reflected by B towards F. The segments DE and BC of the circuit go through tubes containing water flowing with the velocity V. According to Fresnel's formula, the difference between the velocities of the transmitted beam and of the reflected beam is

$$v_T - v_R = 2(1 - 1/n^2)V. \qquad (2.3)$$

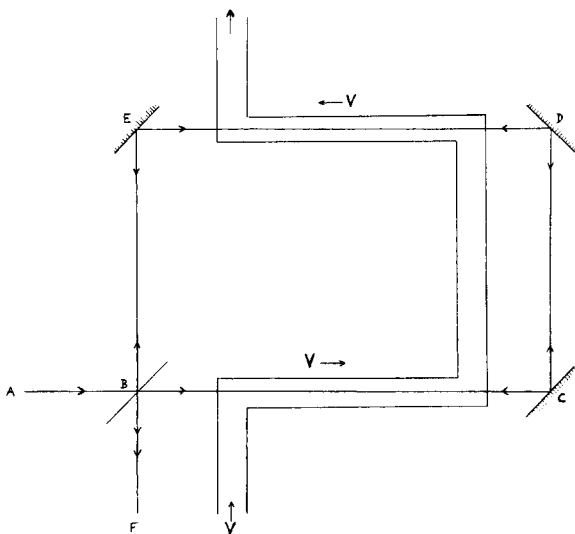Thus the two beams arrive at F out of phase and interfere.

Fig. 1. Fizeau experiment.

Fizeau measured the shift of the interference fringes when the velocity V is inversed, and obtained a result in agreement with (2.3) (a more complete discussion of the experiment may be found in Ref. 2).

The result of Fizeau was corroborated, after the advent of special relativity, by a series of experiments performed by Zeeman (1914), who claimed to be able to measure the dispersive correction to formula (2.1) (this claim has recently been disputed by Lerche[2]).

Thus Fresnel's aether-drag law was, well before the era of relativity, amply demonstrated by experiment. However, it remained a purely phenomenological law, there being no satisfactory theory for it.

The reason for this is easy to see. The propagation of light in a moving medium is (in classical terms) a dual phenomenon, partly optical and partly mechanical. This duality can be fully resolved only in the relativistic quantum theory. In the nonrelativistic (Galilean) theory, even though we can associate matter waves to particles, these are of a different nature from light waves. Galilean light waves in a given homogeneous medium and in a given direction can be characterized by their universality, their velocity v being independent of the place and time where they are observed; but this *kinematical* statement is simply not sufficient to determine the dependence of v on the density *or* on the velocity V of the medium. This can only be determined by a *dynamical* theory of the propagation of light in the medium.

Before discussing, in Sec. IV, such a dynamical theory of Fresnel's law, we shall first review its relativistic kinematical interpretation.

III. FIZEAU'S EXPERIMENT AS A TEST OF SPECIAL RELATIVITY

The relativistic derivation of Fresnel's law is due to Laub and von Laue (1907). According to the principle of relativity, the velocity v' of light relative to the proper frame (R') of a transparent medium (the Galilean frame in which the medium is at rest) depends only on the medium. Thus

$$v' = c/n. \tag{3.1}$$

It then follows from the relativistic law of composition of parallel velocities that the velocity relative to the laboratory [frame (R)] of longitudinal light waves (propagating in the direction of the relative velocity V) is

$$v = \frac{v' + V}{1 + v'V/c^2} = \frac{c/n + V}{1 + V/nc}. \tag{3.2}$$

Expanding this exact formula in powers of V/c yields

$$v = c/n + (1 - 1/n^2)V - (1 - 1/n^2)V^2/cn + \cdots, \tag{3.3}$$

which reduces, in the nonrelativistic approximation ($V \ll c$), to Fresnel's first-order formula (2.1).

The result of Fizeau's experiment is thus explained, but this does not yet allow us to consider this experiment as a test of special relativity. In order to do so we should compare the relativistic formula (3.2) with the corresponding Galilean formula. However, as already mentioned, there is no unique Galilean kinematical theory for this phenomenon.

It is tempting to assume that, in any case (relativistic or Galilean), the proper velocity v' is always equal to c/n. But, as this is really a consequence of the principle of relativity, the experiment then only tests the internal consistency of special relativity. Furthermore, in the limit of a medium of vanishing density ($n \to 1$), this assumption would give (according to the Galilean law of addition of velocities) $v = c + V$, which contradicts the Galilean principle of the universality of the velocity of light relative to absolute space.[4]

Anticipating Sec. IV, let us now assume that we have a (dynamical) Galilean theory of the propagation of light in a transparent medium. As mentioned in Sec. I, there is another, more fundamental, reason why the the experiment cannot discriminate between the Galilean and the relativistic theories.

The comparison of the Galilean and Lorentz transformations shows that all mechanical relativistic effects are of the second order in V/c. While this argument does not apply directly to optics, it is well known that any *measurable* difference between the aether (Galilean) theory and the relativistic theory of the propagation of light in vacuum can only be of the second order in V/c. Thus we anticipate that the relativistic part of the Fizeau effect, which is partly mechanical and partly optical, should also be of the order of V^2/c^2, which is well below the sensitivity of the Fizeau experiment.

IV. MICROSCOPIC DERIVATION OF THE FRESNEL FORMULA

The microscopic theory of the propagation of light in matter was first worked out by Lorentz,[7] as a consequence of his microscopic electromagnetic theory. Without going into the details of this theory, I wish to point out that the mechanical part of Lorentz's original theory was Galilean, with special effects superimposed in order to recover Maxwell's macroscopic equations.[4] As these effects (the Lorentz contraction of lengths, the dependence of mass on velocity, etc.) were of the order of V^2/c^2, they do not come into play in Lorentz's derivation of the Fresnel formula, which was thus fundamentally Galilean (this point seems to have escaped Einstein, who was of the opinion that, because the electrodynamics of Maxwell–Lorentz anticipated relativity, the success of Lorentz's explanation of the Fresnel

formula did "*not in the least diminish the conclusiveness of the experiment . . . in favour of the theory of relativity*"[3]).

I shall consider here a simplified one-dimensional quantum model of the propagation of light in a refractive medium, basically equivalent to Lorentz's theory, but which has the advantage of being purely mechanical. An idealized crystal is made up of evenly spaced pointlike atoms. When a photon, traveling in vacuum at the velocity c, encounters an atom, it is absorbed by that atom, and another photon (of the same wavelength) is emitted after a time lag τ (in other words, the scattered wave is out of phase with the incident wave). This process may be represented by the space-time diagram (Fig. 2), drawn in the laboratory frame (R).

Qualitatively, this model explains quite simply the Fresnel partial drag effect. A light signal traveling through the crystal is a composite, part photon (wavy line), part excited atom (full line). It is in effect the atom part of the signal which is "dragged" along by the motion of the crystal.

It is a simple exercise in geometry to compute the velocity v of the composite signal, given by the inverse slope of the dashed line, in terms of the velocity c of the photons, the velocity V of the atoms, the time lag τ, and the spacing l between atoms (all these quantities being measured in the laboratory). The result is

$$v = c \frac{1 + (V\tau/l)(1 - V/c)}{1 + (c\tau/l)(1 - V/c)}. \qquad (4.1)$$

This computation does not take into account the recoil of the atoms absorbing (or emitting) a photon. Under the extreme assumption that each atom recoils independently, the recoil is negligible if, in the case $V = 0$, the atom of mass m absorbing a photon of wavelength λ recoils during the time τ along a distance (computed in the nonrelativistic approximation)

$$\Delta l = h\tau/\lambda m \ll l \qquad (4.2)$$

(h being the Planck constant), As $c\tau/l$ is of the order of, or
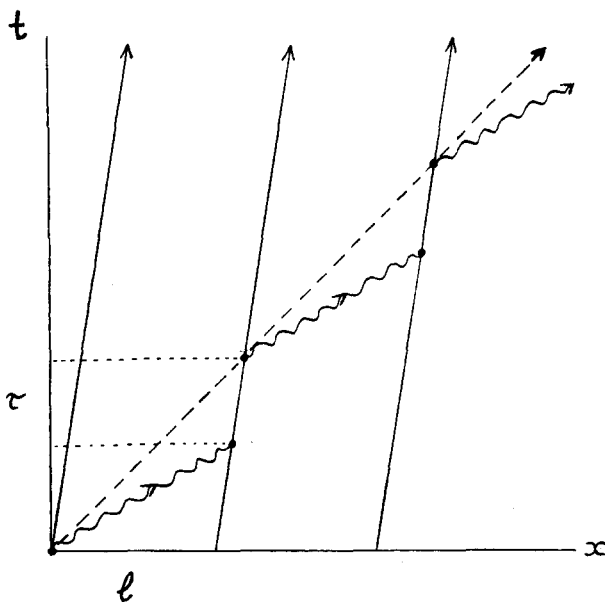

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Fig. 2. Space-time diagram of the propagation of light in a moving crystal (simplified model).

small before, unity [Eq. (4.5) below], this is realized if

$$\lambda \gg \lambda_c = h/mc, \qquad (4.3)$$

which is amply satisfied for visible wavelengths (the Compton wavelength of a typical atom of mass number 20 is $\lambda_C = 0.7 \times 10^{-10}\ \mu$).

The difference between Galilean and relativistic kinematics comes into play when the velocity v of light in the moving crystal, given by Eq. (4.1), is compared to the velocity v_0 of light in the same crystal, this time at rest in the laboratory ($V_0 = 0$):

$$v_0 = \frac{c}{1 + c\tau_0/l_0} = \frac{c}{n}. \qquad (4.4)$$

If Galilean kinematics are assumed, the parameters τ and l are independent of V ($\tau = \tau_0, l = l_0$) so that, according to (4.4),

$$\tau/l = (n - 1)/c. \qquad (4.5)$$

Inserting this into (4.1) yields the Galilean aether-drag formula

$$v_{\text{Gal}} = \frac{c/n + (1 - 1/n)V(1 - V/c)}{1 - (1 - 1/n)V/c}. \qquad (4.6)$$

This formula may be expanded in powers of V/c, giving

$$v_{\text{Gal}} = \frac{c}{n} + \left(1 - \frac{1}{n^2}\right)V - \left(1 - \frac{1}{n}\right)\frac{V^2}{cn^2} + \cdots, \qquad (4.7)$$

which reduces to Fresnel's formula (2.1) in the first order in V/c.

Another consequence of formula (4.6) is that the Galilean proper velocity of light in the moving crystal is *not* c/n, as often incorrectly assumed, but (according to the Galilean law of addition of velocities):

$$v'_{\text{Gal}} = v_{\text{Gal}} - V = \frac{c}{n}\frac{1 - V/c}{1 - (1 - 1/n)V/c}. \qquad (4.8)$$

In accordance with the Galilean principle of relativity, this result may also be derived directly from (4.1) written in the proper frame (R') of the crystal, in which the relative velocity of the crystal is $V' = 0$, and the relative velocity of light in vacuum is $c'_{\text{Gal}} = c - V$.

If now we assume instead relativistic kinematics, the time lag τ and the spacing l are related to the proper time lag τ_0 and the proper spacing l_0 by

$$\tau = (1 - V^2/c^2)^{-1/2}\tau_0, \quad l = (1 - V^2/c^2)^{1/2}l_0, \qquad (4.9)$$

and the relation (4.5) must be replaced by

$$\tau/l = (1 - V^2/c^2)^{-1}(n - 1)/c, \qquad (4.10)$$

which, inserted into (4.1), yields of course the relativistic law of composition of velocities (3.2).

In conclusion, a simplified atomic model of the propagation of light explains Fresnel's aether-drag effect and yields, to first order in V/c, Fresnel's formula, whether Galilean or relativistic kinematics are assumed. The second-order deviation between the Galilean formula (4.7) and its relativistic counterpart (3.3)

$$\Delta v = v_{\text{Gal}} - v = (1 - 1/n)V^2/cn \qquad (4.11)$$

gives, in the case of the original Fizeau experiment, an unobservable relative deviation $\Delta v/v$ of the order of 10^{-16}.

Thus the experimenter cannot, on the basis of the Fizeau experiment alone, decide between Galilean and relativistic kinematics. For the theoretician, however, the contest is clearly won by special relativity, which provides a simpler explanation of the result, relying only on kinematics, without requiring any knowledge of the mechanism of the propagation of light in refractive media.

[1]H. Fizeau, C. R. Acad. Sci. **33**, 349 (1851).

[2]I. Lerche, Am. J. Phys. **45**, 1163 (1977).

[3]A. Einstein, *Relativity: the special and the general theory* (Smith, 1931).

[4]M. A. Tonnelat, *Histoire du principe de relativité* (Flammarion, Paris, 1971).

[5]O. Costa de Beauregard, *Précis of special relativity* (Academic, New York, 1966).

[6]A. T. Fresnel, Ann. Chim. Phys. **3**, 57 (1818).

[7]H. A. Lorentz, *Versuch einer Theorie des elektrischen und optischen Erscheinungen in bewegten Körpen* (Brill, Leiden, 1895).