direction to that of the motion. Things are only paradoxical because one is seduced into thinking that we are dealing with a real physics problem. It is like the old chestnut about the electric field associated with a uniform charge distribution which fills all of space... ah, but that is another problem.

ACKNOWLEDGMENTS

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[1]R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA, 1964), Vol. II, Chap. 18.4; see also P. C. Peters, "Electromagnetic Radiation from a Kicked Sheet of Charge," Am. J. Phys. **54**, 239–245 (1986).
[2]Some of these results are anticipated in T. A. Abbott and D. J. Griffiths, "Acceleration Without Radiation," Am. J. Phys. **53**, 1203–1211 (1985).
[3]Reference 1, Chap. 21.6.

The interstellar traveler

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We investigate the physics of an interstellar journey on board a spaceship with a constant acceleration, in the framework of special relativity. It is in principle possible to cross the Galaxy within a human lifetime. The aspect of the sky seen from the spaceship is severely distorted by relativistic aberration; most of the visible sky shrinks to a small region of strongly enhanced luminance in the direction of motion, leaving the rest of the celestial sphere almost entirely dark. The invisible universe becomes perceptible by the traveler, as the infrared and radio radiations are Doppler-shifted to visible frequencies in the direction of motion. Navigational problems posed by these relativistic effects are examined.

I. INTRODUCTION

The introduction of special relativity at the turn of the century met with considerable resistance, because it shocked common sense. How could time measured in a moving reference frame possibly differ from that measured on Earth? And how could the length of a stick at rest shrink when measured from a moving reference frame? The aspect of the environment clearly changes very drastically when one travels at relativistic velocities, and the purpose of this paper is to describe what an interstellar traveler sees crossing the Galaxy on board a spaceship which experiences a constant acceleration (or deceleration), equal to that of gravity on Earth.

While this exercise has little practical application at present time, for lack of adequate technology, it applies the theory of special relativity to a more realistic case than Mr. Tompkins' relativistic bicycle,[1] and thus provides meaningful insight into the consequences of this theory. It also leads us to examine navigational problems posed by the relativistic velocity and the distorted and blueshifted view of the universe as seen from the spaceship. Finally, by bringing our senses to perceive a strongly distorted and otherwise invisible universe, this journey puts our human existence in a broader perspective.

Various aspects of the present study have already been discussed in the literature. For example, the interstellar journey has been presented by von Hörner[2] and Sagan,[3] the relativistic effects of aberration and Doppler shift by Blatter and Greber[4] and Greber and Blatter,[5] the relativistic deformations of a sphere by Suffern.[6] However, our rederivation of known results and our application of both relativistic effects to the aspect of the whole sky provides for the first time a complete view of interstellar relativistic flights, and reveals unforeseen navigational problems.

Our paper is divided into five parts. We first rederive the equation for the motion of a relativistic spaceship, in order that the paper be self-contained. We apply this equation to a series of destinations within and beyond the Galaxy. We then investigate in greater detail how aberration distorts the large-scale aspect of the night sky, and how the Doppler effect modifies the color and even the nature of the radiations perceived by the human eye. We briefly discuss how the diffuse background radiation (the night sky) is perceived as the spaceship approaches the velocity of light. Finally, we study the navigational problems posed by the distorted aspect of the sky when one travels at a relativistic velocity.

In this paper, we do not discuss the aspect of individual objects seen at a relativistic velocity, for example, the deformation of spherical objects[6] such as planets or globular clusters or the visual rotation of rapidly moving objects;[7] we only deal with the aspect of the whole sky.

II. THE KINEMATICS OF A RELATIVISTIC JOURNEY

First, we derive the equation governing a spaceship's motion in the frame of special relativity.[8] The spaceship has two parts, one of constant mass (the infrastructure and the on-board equipment), and the other of variable mass (the fuel).

A terrestrial observer measures time t in the inertial reference frame $\mathscr{R}$ tied to the Earth. A clock on board the spaceship measures proper time τ in the noninertial reference frame $\mathscr{F}$. At time t in $\mathscr{R}$, the inertial mass of the spaceship is m, its velocity V, and its associated relativistic factor $\gamma = 1/\sqrt{1-\beta^2}$ with $\beta = V/c$; all quantities V, γ, m are time dependent.

The relativistic equation of motion of the spaceship moving under the influence of an external field of force is

$$\left(\frac{dP}{dt}\right)_{/\mathscr{R}} = F^{\text{ext}}, \tag{1}$$

where $P = \gamma m V$ is the total relativistic momentum of the system in $\mathscr{R}$ and F^{ext} the external force.

Between time t and $t+dt$, a quantity of matter dm_G is ejected in opposite sense to the motion; this provides the thrust. The relation between the velocity u of that quantity of matter measured in $\mathscr{F}$ and its velocity U in $\mathscr{R}$, is given by the relativistic transformation of velocities

$$U = \frac{u-V}{1-uV/c^2}. \tag{2}$$

In special relativity, the total mass of a system is generally not equal to the sum of the masses of its components, and the mass loss dm of the spaceship will not be equal to $-dm_G$. Note that, following Taylor and Wheeler,[9] we consider the mass of a closed system as invariant, and dismiss the notion of relativistic mass. In addition to its physical justification, this point of view presents a definite technical advantage for studying open systems in special relativity, as in the present case.

To determine the relation between dm and dm_G, and thus the mass loss, we have to apply the conservation of energy[10] of the system between times t and $t+dt$

$$d(\gamma m c^2) + \gamma_U \, dm_G \, c^2 = 0, \tag{3}$$

where $\gamma_U = 1/\sqrt{1-(U/c)^2}$ is the relativistic factor associated with U.

We obtain the equation of motion by determining the total variation of momentum dP, which is the sum of the variations of the spaceship's momentum and of the momentum carried away by the thrust

$$dP = d(\gamma m V) - \gamma_U \, dm_G \, U. \tag{4}$$

Using Eqs. (1), (2), (3), and (4), we obtain

$$\frac{\gamma}{1-uV/c^2}\left(m\,\frac{dV}{dt} + \frac{u}{\gamma^2}\,\frac{dm}{dt}\right) = F^{\text{ext}}. \tag{5}$$

Next, we consider that the external force is negligible. This is a reasonable approximation for the motion of a spaceship through the interstellar medium. The galactic gravitational field can safely be neglected, since it only becomes significant over time scales several orders of magnitude above those required for crossing the Galaxy with a relativistic spaceship.

In order to cross a distance d in the shortest time (measured by the traveler), the spaceship should always have the highest possible acceleration. But, since human travelers are on board, it is advisable, for reasons of health and comfort, to limit the proper acceleration g of the spaceship to 9.81 m s^{-2}, which corresponds to the acceleration of gravity on Earth; this is the value adopted throughout this paper. Needless to say, while the proper acceleration of the spaceship is constant, its acceleration measured in $\mathscr{R}$ is definitely not. Relativistic transformation of accelerations gives

$$\frac{dV}{dt} = \left(1-\frac{V^2}{c^2}\right)^{3/2} g. \tag{6}$$

Integrating this differential equation twice provides the velocity $V(t)$ and the distance $x(t)$, starting from rest at $t=0$

$$V(t) = \frac{gt}{\sqrt{1+(gt/c)^2}}, \tag{7}$$

$$x(t) = \frac{c^2}{g}\left(\left[1+\left(\frac{gt}{c}\right)^2\right]^{1/2} - 1\right). \tag{8}$$

We now turn to the fuel consumption. Equations (5) and (6) allow us to obtain the rate of mass loss

$$\frac{dm}{dt} = -\frac{g}{u}\, m. \tag{9}$$

For the sake of simplicity, we assume that the velocity u of ejection of matter from the spaceship is constant. In a Newtonian framework, γ is near unity and the system's mass decreases exponentially with time, with a damping factor equal to u/g. If one assumes $u=c$, as in the case of photonic thrust resulting from matter–antimatter annihilation, the damping factor is about one year.

With Eq. (7), and after integration, we obtain

$$m = \frac{m_0}{(gt/c + \sqrt{1+(gt/c)^2})^{c/u}}, \tag{10}$$

where m_0 is the spaceship's mass at the initial time ($t=0$).

Finally, we derive the relation between the time in $\mathscr{R}$ and in $\mathscr{F}$. Because of time dilation, the time $\tilde{T}_F$ (proper time) measured in the accelerated frame $\mathscr{F}$ will be shorter than $\tilde{T}$, the duration in $\mathscr{R}$ (on Earth), or in any other reference frame.

The infinitesimal time intervals between two neighboring points in space are related by

$$d\tau = \frac{dt}{\gamma}.$$

Integrating, one obtains

$$\tilde{T}_F = \int d\tau = \int_0^{\tilde{T}} \frac{dt}{\gamma},$$

and, with Eq. (7),

$$\frac{g\tilde{T}}{c} = \sinh\left(\frac{g\tilde{T}_F}{c}\right). \tag{11}$$

This spectacular equation shows that the time ($\tilde{T}_F$) measured on board the spaceship varies roughly as the logarithm of the time ($\tilde{T}$) measured on Earth. We quantify this result below.

III. JOURNEY THROUGH THE GALAXY AND BEYOND

Interstellar travel at velocities close to that of light should in principle be considered in the framework of our present scientific and technical knowledge. But it is not the purpose of our paper to discuss the feasibility of such travels; we optimistically assume that there are no insuperable obstacles to building a relativistic spaceship, and that technological solutions will be found in the future for transporting the requested amount of fuel (or gathering it from the interstellar medium on the way), for transforming it efficiently into energy, and for shielding the spaceship from small obstacles. We refer to von Hörner[2] and Sagan[3] for further details on the technical feasibility of interstellar flights.

We consider the journey in two stages, one of constant acceleration to mid-journey, and the other of constant deceleration to the destination. The deceleration can be obtained by reversing the spaceship's orientation, and thus the sense of acceleration. The constant thrust from the spaceship's motor creates an artificial gravity which avoids the problems of traveling in weightlessness. With such a travel plan, a spaceship can make galactic, and even extragalactic, journeys within a human lifetime.

The equations of Sec. II can very simply be used for our flight plan. We successively derive $\gamma_{\max}$, the relativistic factor at mid-journey, when the spaceship's velocity is maximum, T_F, the duration of the journey measured on board (in $\mathscr{F}$), T, its duration measured by an observer at rest in a frame tied to the Earth (in $\mathscr{R}$), and $\mathscr{M}$, the ratio of initial to final mass of the spaceship

$$\gamma_{\max}=1+\frac{gD}{2c^2}\simeq 1+0.516\left(\frac{D}{\text{l.y.}}\right)\left(\frac{g}{g_0}\right), \tag{12}$$

$$T_F=\frac{2c}{g}\cosh^{-1}\gamma_{\max}\simeq 1.937\left(\frac{g_0}{g}\right)\ln(2\gamma_{\max})yr, \tag{13}$$

$$T=\frac{2c}{g}\sqrt{\gamma_{\max}^2-1}\simeq 1.937\left(\frac{g_0}{g}\right)+\left(\frac{D}{\text{l.y.}}\right)yr, \tag{14}$$

$$\mathscr{M}=[\gamma_{\max}+\sqrt{\gamma_{\max}^2-1}]^{2c/u}\simeq[2\gamma_{\max}]^{2c/u}, \tag{15}$$

where D is the total distance to be traveled, g the value of the constant acceleration (or deceleration) of the spaceship, $g_0=9.81$ m s^{-2} the mean acceleration of Earth's gravity, $\beta_{\max}$ the maximum value of V/c, and l.y. stands for lightyears.

The durations are the same in the two (accelerated and decelerated) stages of the journey, $T=2\tilde{T}$, but the mass consumption is not; this explains the factor 2 in Eqs. (13) and (14), as well as the exponent 2 in Eq. (15).

The asymptotic values are valid for large $\gamma_{\max}$, that is for long-range flights. Note that relativistic velocities ($\geqslant 0.1c$) are only reached after $D\simeq 0.01$ lightyears, or about 16 times the distance from Earth to Pluto, and that relativistic flights at constant and moderate acceleration cannot take place over distances as short as the size of the solar system. This is an important remark for those who were hoping to make relativistic flights within the solar system.

Table I gives the on-board duration of the journey for various destinations and for $g=g_0$, as well as the mass ratios for a velocity of ejection equal to c. For an observer on Earth, the duration of the journey rapidly becomes equal to the distance in lightyears plus two years. This is easily understood, since the spaceship's velocity is almost always close to that of light.

It is the purely relativistic phenomenon of time dilation that allows interstellar travels within human lifetime; thanks to the logarithmic relation (11) between T and T_F, less than 10 or 20 years are necessary to reach any destination within the Galaxy. In a Newtonian view of the world, one could even cross the universe in 45 years! The catch is the mass ratio $\mathscr{M}$; for the closest destination it is already almost 40, and increases as the square of the distance. For comparison, the mass ratio of an Airbus 320 is about 1.3, and that of the Ariane launcher about 10 (but see Ref. 11). For extragalactic destinations, so much fuel is needed that the spaceship becomes comparable in size to a minor planet. Perhaps part of the solution is to use matter at nuclear densities, such as a neutron star, or to gather the fuel from the interstellar medium on the way.

Table I. A series of destinations for a trip made with uniform acceleration and deceleration of $1g$; the distances are in lightyears and the durations in years.

Destination	Distance	Duration on Earth	Duration on-board	Mass ratio
Proxima	4.2	5.82	3.53	38.15
Sirius	8.7	10.46	4.63	118.7
Tau Ceti	11.9	13.70	5.14	202.3
Altair	16.1	17.93	5.66	345.1
Vega	26.5	28.37	6.54	860.9
Arcturus	36	37.89	7.10	1 534
Regulus	76	77.91	8.50	6 481
Spica	260	261.9	10.85	73 244
Betelgeuse	380	381.9	11.58	155 740
Rigel	660	661.9	12.64	467 846
Deneb	1 500	1 502	14.23	2 409 049
Gal. Center	30 000	30 002	20.03	9.62×10^9
Cross Gal.	100 000	100 002	22.36	10.7×10^9
Magellanic C.	165 000	165 002	23.33	29.1×10^9
Andromeda	2.25×10^6	2.25×10^6	28.39	5.4×10^{12}
Virgo cluster	36×10^6	36×10^6	33.76	1.39×10^{15}
Edge of universe	15×10^9	15×10^9	45.44	2.41×10^{20}

We now describe what a traveler would see halfway through the journey, when the spaceship's velocity is maximum. The term *forward* refers to directions close to that of motion, and *backward* to the opposite direction. Two relativistic effects modify the visual aspect of the universe, aberration, and Doppler effect.

IV. ASPECT OF THE SKY DISTORTED BY RELATIVISTIC ABERRATION

Aberration is the apparent deviation of the direction in which a source is observed when it moves with respect to the observer. This is a simple consequence of the transformation of velocities, and also occurs at low velocities. Our common experience is that of raindrops which fall vertically when we are motionless, but hit the windshield more than the back window of the car when it moves fast.

The transformation of the wave four-vector[12] leads directly to the equations

$$\sin\theta=\frac{1}{\gamma_{\max}}\frac{\sin\theta'}{1-\beta_{\max}\cos\theta'},\qquad \cos\theta=\frac{\cos\theta'-\beta_{\max}}{1-\beta_{\max}\cos\theta'} \tag{16}$$

where θ and θ' are the angles between the *forward* direction and that in which the source is observed, in $\mathscr{R}$ and $\mathscr{F}$, respectively. The directions $\theta'=0$ and $\theta'=\pi$ are the only ones not affected by aberration.

We consider a distribution of stars on the celestial sphere which is uniform in $\mathscr{R}$, with a density ρ per unit solid angle. How does this distribution appear to a traveler inside the spaceship? Let $\rho'(\theta')$ be the density of stars measured in $\mathscr{F}$. The conservation of the total number of stars in the two reference frames implies

$$\rho\, d\Omega=\rho'\, d\Omega'. \tag{17}$$

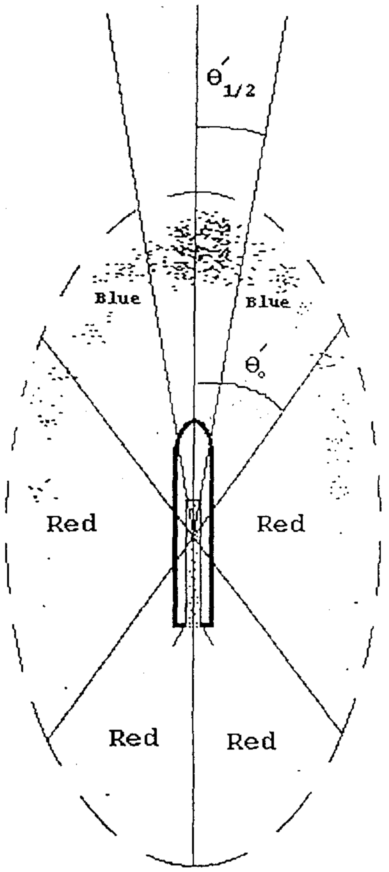

Fig. 1. The sky observed from the spaceship is affected by a combination of aberration and Doppler effect.

With $d\Omega = 2\pi \sin\theta\, d\theta$, $d\Omega' = 2\pi \sin\theta'\, d\theta'$, and, using Eq. (16), this equation becomes

$$\rho'(\theta') = \frac{1}{\gamma_{\max}^2 (1 - \beta_{\max} \cos\theta')^2}\, \rho. \tag{18}$$

This angular density profile is maximum for $\theta' = 0$, and decreases to a minimum at $\theta' = \pi$. The higher the velocity, the more this profile peaks in the *forward* direction.

To quantify this result, we determine the half-aperture $\theta'_{1/2}$ of the cone containing half the stars in the sky (see Fig. 1). At rest in $\mathscr{R}$, this cone corresponds to one celestial hemisphere, and $\theta_{1/2} = \pi/2$. Integrating Eq. (17) over half the stars in the sky, we obtain

$$2\pi\rho = \int_0^{\theta'_{1/2}} \rho'(\theta')\, d\Omega.$$

The solution of this equation is

$$\cos\theta'_{1/2} = \beta_{\max}. \tag{19}$$

The aperture of the cone containing half the stars in the sky indeed tends to zero as the velocity tends to c.

During an interstellar relativistic flight, the stars which at rest uniformly cover the celestial sphere, all seem to converge to the point toward which the spaceship is heading (see Fig. 2).

To give a more precise idea of how aberration distorts the aspect of the sky, we have plotted a grid of coordinates on the celestial sphere, analogous to those used for the Earth: meridian lines which run north–south from the poles, and parallel lines which run east–west across the sphere [Fig. 3(a)]. These lines are severely distorted when plotted in the frame $\mathscr{F}$ of the relativistic spaceship, as shown in Fig. 3(b), the apparent angular size of extended objects (thus the resolution) is reduced and the two celestial poles are moved into the field of view by aberration.

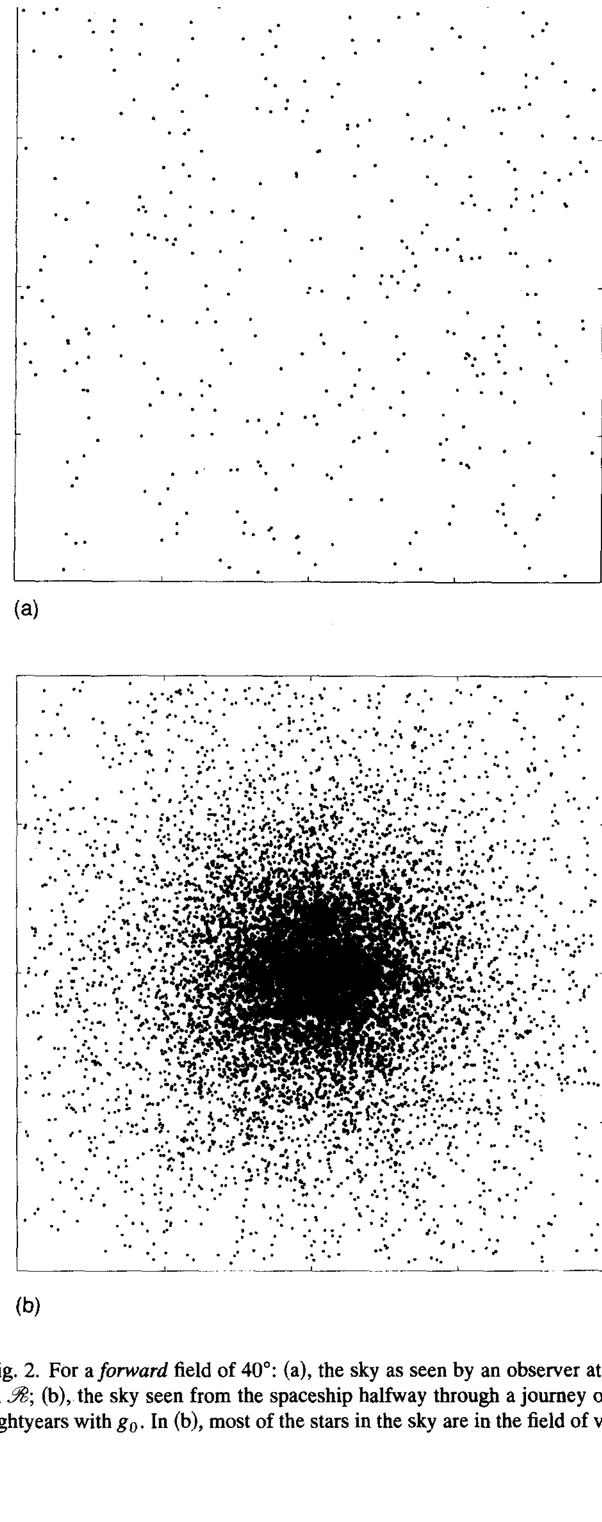
(a)
(b)

Fig. 2. For a *forward* field of 40°: (a), the sky as seen by an observer at rest in $\mathscr{R}$; (b), the sky seen from the spaceship halfway through a journey of 15 lightyears with g_0. In (b), most of the stars in the sky are in the field of view.

These distortions have nothing to do with the deformations of space–time near massive objects in general relativity; they are only geometric effects which can be corrected

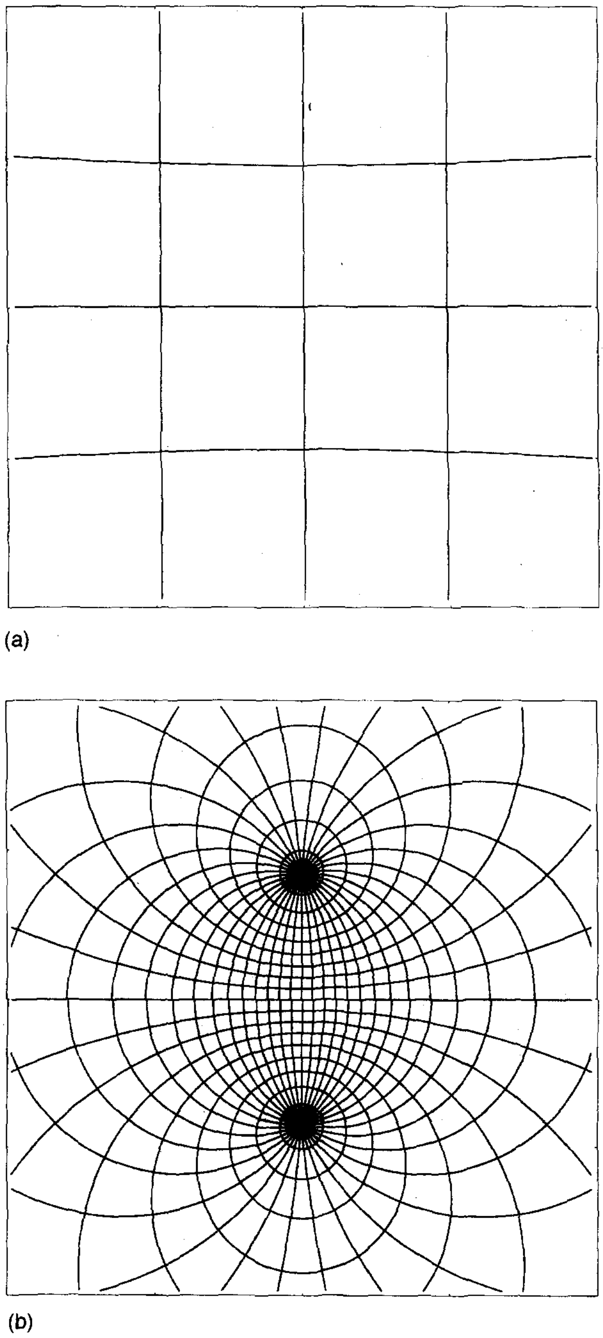

Fig. 3. The system of meridian and parallel lines plotted on the celestial sphere; (a) in frame $\mathscr{R}$ at rest, and (b) for a velocity equal to $0.9c$. At relativistic velocities, the two celestial poles are drawn into the field of view.

for [using Eq. (16)], given the spaceship's velocity. We remind the reader that, in special relativity, the structure of space–time is pseudo-Euclidean.

V. ASPECT OF THE SKY SHIFTED BY DOPPLER EFFECT

The Doppler effect is the spectral shift of electromagnetic waves measured by an observer who is not at rest with respect to the source of the waves. The equation for the Doppler shift is

$$\frac{\nu'}{\nu}=\gamma_{\max}(1+\beta_{\max}\cos\theta), \tag{20}$$

where ν is the frequency of the waves in $\mathscr{R}$ and ν' their frequency in $\mathscr{F}$.

Or, using Eq. (16)

$$\frac{\nu'}{\nu}=\frac{1}{\gamma_{\max}(1-\beta_{\max}\cos\theta')}. \tag{21}$$

In the spaceship's frame of reference (see Fig. 1), the waves are shifted toward higher frequencies (blueshifted) for θ' between 0 and θ'_0 such that

$$\cos\theta'_0>\frac{\sqrt{\gamma_{\max}-1}}{\sqrt{\gamma_{\max}+1}}. \tag{22}$$

In the other directions, the shift is toward lower frequencies (redshift). Conversely, the visible window falls in the range of lower energies in the *forward* direction, and in that of higher ones in the *backward* direction.

During an interstellar relativistic flight, and as the velocity increases, the human eye successively perceives the infrared, millimetric, and radio sky forward.

It would also successively perceive the ultraviolet and x- and γ-ray sky in the opposite direction, if its luminance were not below the detection limit of the eye.

Table II gives, for various destinations, the values of $\gamma_{\max}$, $\theta'_{1/2}$ (half-angle of the cone containing half of all the stars), θ'_0 (angle at which there is no Doppler shift), as well as the range of frequencies that becomes visible in the *forward* direction. The limits of the visible range are blue and red. Relativistic effects are noticeable even for close destinations; in a journey to Deneb, for example, it is the cosmic microwave background that becomes visible *forward.*

Stars were used for convenience to describe how aberration distorts the aspect of the sky. But in fact they soon disappear from view as the radio sky is Doppler shifted into the visible domain. A completely different scenery appears,

Table II. Parameters illustrating the two relativistic effects for various destinations.

Destination	Distance	$\gamma_{\max}$	$\theta'_{1/2}$	θ'_0	Forward radiation	
Proxima	4.2	3.17	18.4°	43.8°	2.5–4.3 μm	ir
Altair	16.1	9.31	6.17°	26.1°	7.4–13 μm	ir
Betelgeuse	380	197	17′	5.77°	160–280 μm	far ir
Deneb	1 500	775	4.4′	2.9°	0.62–1.1 mm	millimeter
Accross Gal.	100 000	51 600	4.1″	21′	40–70 mm	millimeter
Andromeda	2.25×10^6	1.16×10^6	0.18″	4.5′	0.93–1.6 m	meter
Virgo cluster	36×10^6	18.6×10^6	0.01″	1.2′	15–26 m	radio

Table III. Some reference luminances.

Luminance (cd m^{-2})	Source
1 600 000	Sun
1 000 000	Dazzle limit
160 000	Electric arc
30 000	White paper in the sun
2 500	Full moon
3	Photopic limit (color)
10^{-3}	Scotopic limit (gray)
2×10^{-6}	Extragalactic night sky
10^{-6}	Limit of perception

pointlike extragalactic radiosources against a bright diffuse background of synchrotron radiation from electrons in our Galaxy. Some extragalactic radio sources are known to be very extended, but they will appear pointlike because of aberration.

VI. LUMINANCE AND COLOR OF THE SKY SEEN FROM A RELATIVISTIC SPACESHIP

Let us now describe in somewhat more detail the appearance of the sky background viewed from the relativistic spaceship. Forward, its luminance is enhanced by aberration, and shifted to lower frequencies by the Doppler effect. In the backward direction, the sky is completely black.

We first introduce the appropriate SI unit for the luminance L of the sky background, the candle per square meter (cd m^{-2}), which refers to the luminance at the reference frequency of 540×10^{12} Hz. The eye is most sensitive at this frequency in daylight; for night vision, the maximum sensitivity of the eye is shifted 48 nm toward the blue.

$$1\ \mathrm{cd\ m^{-2}} = \frac{1}{683}\ \mathrm{W\ m^{-2}\ sr^{-1}}.$$

In order to make the cd m^{-2} unit meaningful, we give in Table III the luminance of a few familiar objects, and the limits of sensitivity of the eye. The photopic limit is the luminance above which the eye perceives colors, and the scotopic limit is that below which the eye only perceives shades of grey. There is a range of 3 orders of magnitude in between, where some subjects perceive colors, others do not. Other parameters, such as the luminosity contrast, affect the way the eye perceives extended sources of light, but they will be ignored here.

At rest in $\mathscr{R}$, the luminance of the sky background is assumed to be spatially uniform (this is not quite correct, because it is enhanced at all frequencies in the direction of our galactic disk), and equal to 2×10^{-6} cd m^{-2}. One effect of aberration is to enhance the luminance of the sky background in the direction of motion, because the angle $\theta'_{1/2}$ [Eq. (19)] becomes smaller; the luminance of the sky as seen from the spaceship is thus no longer uniform.

The luminance in the forward direction at mid-journey is given by the solid line shown on Fig. 4 for different destinations, whose distance is given by the abscissa. This line has been computed with Eq. (18) under the assumption that the sky background has the same luminance at all frequencies. Horizontal dashed lines indicate reference luminances; it is interesting to note that they are roughly equally spaced in logarithmic units.

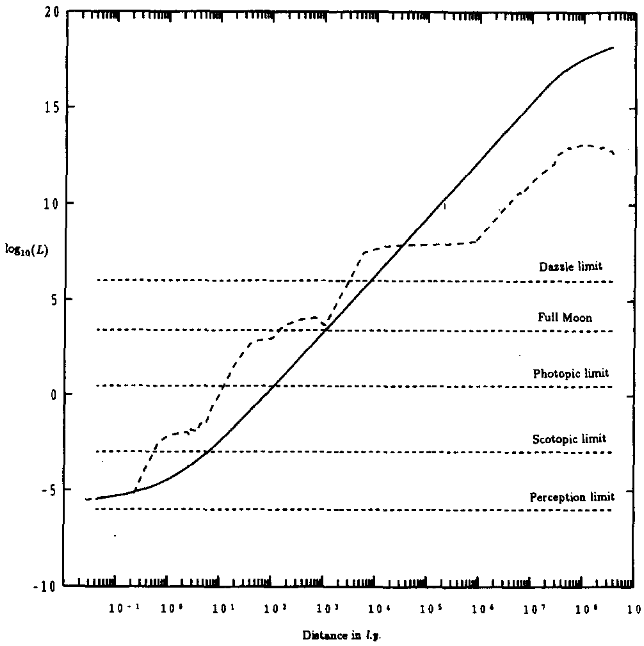

Fig. 4. Logarithm of the luminance L (in cd m^{-2}) of the sky background, observed from the spaceship in the direction of motion, as a function of the distance to the goal of the journey; the dashed broken line shows how the frequency dependence of the sky background affects its luminance.

For the shortest journeys, the interstellar traveler perceives only a greyish sky at mid-journey. Beyond 100 lightyears, the visible sky becomes a colored spot with an apparent diameter of about 2 degrees [this angle is computed with Eq. (19)]. For a journey of 1000 lightyears, the sky is comparable to the moon in luminance, but has half its size. Beyond 7000 lightyears, the whole sky becomes black, except for a dazzling point source.

But the luminance of the sky background depends on frequency. It is approximately the same in the visible and centimetric domains, slightly brighter in the infrared, but 5 orders of magnitude fainter in the X and γ ranges, and 7 orders of magnitude fainter at lower radio frequencies. Thus, if we also take into account the effect of Doppler shift on the appearance of the sky background, its luminance is more closely represented by the dashed broken line on Fig. 4. Note that this dashed line ignores differences between the galactic and extragalactic components of the sky background.

The dashed broken line is not markedly different from the solid line; thus taking into account the frequency dependence of the sky background (and probably its spatial dependence as well) does not change the picture qualitatively, because aberration is the dominant effect.

On the other hand, the exact color that the traveler perceives depends critically on the exact frequency dependence of the sky background. Such a discussion is outside the scope of the paper; all we can say is that the luminance gradient is predominantly negative for most destinations within the Galaxy, and thus that the color of the sky is rather blue or violet during most of the journey.

VII. NAVIGATIONAL PROBLEMS POSED BY THE RELATIVISTIC EFFECTS

This very altered view of the environment and the high velocity both pose problems for navigating on-board a relativistic spaceship. It is very difficult to pilot a supersonic jet in real time when it flies at more than 300 m s^{-1}. And this is still a million times slower than the velocity of light. At relativistic velocities, there is little time for avoiding obstacles, and navigational errors may have catastrophic consequences.

We first consider the hazard of collision with stars or asteroids. Planets are assumed to exist only close to stars. There are about 200 billion stars in our Galaxy. The number of asteroids, a few km across, predicted by the theories of star formation is very model dependent; let us guess that they are 10^{10} times more numerous than stars. To compute the probability of an encounter with such bodies, we calculate their surface projected on the sky: the probability of encounter P is simply the total surface covered by these bodies divided by the total surface of the sky.

$$P \simeq 10^{-14} n D R^2, \tag{23}$$

where n is the density of objects per unit lightyear, D the distance to be traveled in lightyears, and R the radius of the star or asteroid, in units of the sun's radius ($R_\odot$).

The probability of encountering a star while crossing the whole galactic disk ($D=10^5$ l.y., $n \simeq 1$ l.y.$^{-3}$, $R=R_\odot$) is one in a billion, the same as that of encountering an asteroid ($n \simeq 10^{10}$ l.y.$^{-3}$, $R \simeq 10^{-5} R_\odot$). Encounters with large bodies thus do not constitute a hazard for interstellar travelers. On the other hand, the spaceship must be shielded against hydrogen atoms in the interstellar medium (one per cm^3).

Navigational errors can be expected, either at takeoff, or during the journey (gravitational deflection by massive objects). The difficulties in correcting for such errors are of three kinds. First, the higher the spaceship's velocity, the more difficult path corrections are. Such corrections are obtained by applying an acceleration perpendicular to the initial path of the spaceship. This acceleration is g in a frame in uniform and rectilinear motion, coinciding with the spaceship as it is about to turn. We apply the relativistic equations for the addition of accelerations to calculate the acceleration a in frame $\mathscr{R}$ linked to Earth.

$$a = \frac{g}{\gamma^2} \tag{24}$$

and the radius of curvature R_c of the new trajectory is

$$R_c = \frac{c^2}{g} (\gamma^2 - 1) \tag{25}$$

$$\simeq 0.969 \left(\frac{g_0}{g} \right) \gamma^2 \text{ l.y.}, \tag{26}$$

where γ is the relativistic factor tied to the spaceship at the moment of the path correction. In fact, at mid-journey, the turning radius is always larger than D, the total distance.

Second, relativistic aberration decreases the nominal accuracy of navigation. Because of the finite resolution of the navigational instruments, there will be an initial angle of deviation $\delta\theta$ (measured in $\mathscr{R}$) with respect to the direction of the goal. We obtain the angle of deviation $\delta\theta'$ in the spaceship by expanding Eq. (16) for small angles

$$\delta\theta' \simeq \frac{\delta\theta}{2\gamma}. \tag{27}$$

Acceleration to a relativistic velocity thus increases the cone of uncertainty around the goal.

Finally, the luminance and color of the whole scene are completely modified by Doppler effect. In order to continue seeing the goal, assuming that it only radiates in the visible range, one has to use x- or γ-ray telescopes when traveling at a relativistic velocity.

Traveling in interstellar space at relativistic velocities is not as easy as video games let us imagine; our investigations show that there are many subtleties, not to be overlooked.

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[8] Our understanding of the theory of special relativity is best reflected in the textbook by J. Ph. Pérez, N. Saint Cricq-Chéry, *Relativité et quantification* (Masson, Paris, 1986).
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[10] Recall that the energy of a point mass m and of relativistic factor γ is: $E=\gamma m c^2$.
[11] The mass ratio of 10 for Ariane might be misleading; one should keep in mind that the mass loss includes the motors of the lower stages of the launcher, not just the fuel.
[12] J. L. Synge, *Relativity: The Special Theory* (North-Holland, Amsterdam, 1979), Eq. 132, p. 146.

SCIENTIFIC SNOBBERIES

Modern science has its own snobberies—biologists pay more attention to genes than to bunions, and physicists would rather study proton–proton collisions at 20 trillion volts than at 20 volts. But these are tactical snobberies, based on judgments (right or wrong) that some phenomena turn out to be more revealing than others; they do not reflect a conviction that some phenomena are more important than others.

Steven Weinberg, *Dreams of a Final Theory* (Pantheon Books, New York, 1992), p. 11.