Comments on the Hafele–Keating Experiment

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Salient features of the Hafele–Keating test of relativistic clock-rate changes are presented; these changes were for cesium atomic clocks which had been flown around the world, once eastward and once westward, on approximately equatorial paths. An inconsistency for an earth observer, between equal energy but different time rates for eastward and westward moving clocks, is shown to be removed if the Sagnac effect is taken into account in the synchronization of clocks in the earth reference system. Also, it is shown that an atomic clock's directional relativistic loss (gain) for an eastward (westward) circumpolar journey is velocity independent. The observed relativistic time effects may be derived as general-relativistic changes, leaving the question of special-relativistic kinetic effects on a macroscopic clock an open one empirically.

I. INTRODUCTION

Probably no other topic in elementary physics raises as much student interest and questioning as does the relativistic transformation of clock times; also, I think, no other topic has been the center of so extensive a discussion and controversy in the physics literature of recent decades. For those reasons the 1971 Hafele–Keating observations[1] of relativistic time changes in moving clocks is, I believe, one of the important achievements in relativistic physics. The experiment gives the first demonstration of relativistic time change for a macroscopic clock; in contrast, previous confirmations have been for photons (as in the Ives–Stilwell[2] and Pound–Rebka[3] experiments, for special and general relativistic transformations, respectively) or for mesons, with observation of the (special) relativistic transformation of their decay times by means of counting the meson decay products.[4]

In this paper I want, after reviewing the Hafele–Keating experiment, to give a discussion of two aspects of it that have been briefly discussed elsewhere: (1) a relation between the Hafele east-west time asymmetry and the Sagnac effect; and (2) the general-relativistic elucidation of the observed clock-rate changes.

II. THE EXPERIMENT: THEORY AND RESULTS

In October 1971 J. C. Hafele and R. E. Keating, using commercial jet aircraft, flew a set of four atomic clocks on two approximately equatorial around-the-world trips, once eastward and once westward. The elapsed clock times were compared with the standard mean elapsed time that was given by the set of similar atomic clocks maintained by the United States Naval Observatory in Washington. The clocks used were portable cesium atomic clocks whose rate is determined by the hyperfine transition of ^{133}Cs, $\nu = 9\ 192\ 631\ 770$ Hz, now by definition the defining standard for the second. Further clock details may be found in papers cited in Ref. 1.

The theory of the experiment was developed by Hafele,[5] and here I shall only repeat his salient points. The time intervals of moving clocks are calculated relative to intervals for clocks in a hypothetical coordinate system S. This system has the same associated gravitational field as does the earth and has the same orbital and galactic motions, but does *not* have the axial rotation of the earth. The calculation is first made for a clock at rest on the earth's surface, at some point on the equator, and hence with speed $R\Omega$ relative to S, where R is the earth's radius and Ω its axial angular rotation speed. For a clock moving equatorially in a jet plane with speed v relative to the earth, where v is $(+)$ for eastward and $(-)$ for westward motion, the speed relative to S is $R\Omega+v$; time interval is also calculated for this clock, relative to that for S clocks. Elimination of the hypothetical S-clock interval between the equations for rest and moving earth clocks leads to:

$$\Delta t = \Delta t_0(1 - v^2/2c^2 - vR\Omega/c^2), \quad (1)$$

where Δt is time interval for a "flying clock" and Δt_0 is for a rest earth clock. Equation (1) may be regarded—although with certain reservations to be discussed later—as coming simply from the special relativistic equation, $t' = t(1 - w^2/c^2)^{1/2}$, where w is $R\Omega$ for calculation of Δt_0 and is $R\Omega + v$ for calculation of Δt; expansion terms of powers higher than the first in c^{-2} have been dropped in the calculation.

Also, however, the flying clock is at an altitude h relative to the rest clock on the earth, and therefore subject to a general relativistic rate increase factor, gh/c^2, where g is taken as approximately constant for the h values involved. With addition of the altitude term, Eq. (1) becomes:

$$\Delta t = \Delta t_0(1 - v^2/2c^2 - v\Omega R \cos\lambda/c^2 + gh/c^2). \quad (2)$$

One further modification has also been included in Eq. (2): For motion of the clock at latitude λ rather than equatorially a $\cos\lambda$ factor must be included with R. (The rest clock-rate, in contrast, behaves as if slowed by the motion $R\Omega$ at any latitude, since slightly smaller R at larger λ gives a gravitational clock-rate decrease which approximately compensates for the $\cos\lambda$ speed decrease. That is, as Hafele points out,[6] the earth tends toward an "equal time" surface.)

Equation (2) is the one which Hafele and Keating tested, Δt for the clocks flown around the world being compared with the mean elapsed time Δt_0 of the Naval Observatory clocks. It has the striking feature that, since $R\Omega$ is of the order of 1700 km/h, and jet aircraft travel at speeds $v \simeq 1000$ km/h, for westward $(-v)$ travel the kinetic effect will generally give $\Delta t > \Delta t_0$. The contribution of the gh/c^2 term is, of course, always >0, and for jet-plane travel gives an effect comparable to that of the other terms. Details of the flight observations and data reduction are given in Ref. 1. The overall results were in good confirmation of Eq. (2). For the eastward flight the predicted relativistic time difference, with error estimate, was -40 ± 23 nsec; the mean observed value for the set of four clocks was -59 ± 10 nsec, the second number being the standard deviation. For the westward flight the comparable figures are: predicted, $+275 \pm 21$; mean observed, $+273 \pm 7$.

III. EAST-WEST ASYMMETRY, SAGNAC EFFECT

In the hypothetical non-rotating coordinate system S the rate difference between eastward and westward moving clocks is readily understood, for the flight velocity v is added to or subtracted from the earth-surface rotational motion $R\Omega$. For an earth rest observer there is, however, the following puzzling feature. Suppose two similar clocks of mass m are in motion on the equator, with speed v relative to the Earth, one to the east and one to the west. (For simplicity we will take $h = 0$.) For each, the kinetic energy must be $E = mc^2(\gamma - 1)$, where $\gamma = (1 - v^2/c^2)^{-1/2}$. The earth observer should, therefore, also expect to find the same relativistic time change for each clock, $\Delta t = \Delta t_0/\gamma$, since energy and time are comparable components of conjugate 4-vectors. In fact, however, the observationally confirmed Eq. (2) tells us otherwise.

The apparent inconsistency is removed by considering the role of the Sagnac effect in a synchronized set of earth clocks with which the flying clocks could be compared. In that system we now show the east-west asymmetry disappears and, although there is a necessary discontinuity at the point of closure of a latitudinal system of synchronized clocks, there is a comparable discontinuity between the clocks flown east and those flown west around the Earth. Also, it becomes apparent that terrestrial circumpolar motion has a velocity-independent effect on atomic clocks.[7]

First, we recall that the Sagnac effect[8] requires that if light is reflected around a planar closed path, area A, in a system rotating about an axis normal to A with angular speed Ω, the light traversal time is altered by

$$\delta t = \pm 2A\Omega/c^2, \quad (3)$$

where the sign is positive if the sense of rotation and light path are the same, and negative if opposite.

For the limiting case of a circular light path, radius R, on a body rotating with speed Ω, in an inertial system K, Eq. (3) can readily be seen as coming from a simple, approximate calculation. Say the light leaves from a point a on the rotating body, moving in the same sense as that of Ω, and returns to a in time t. The light has travelled $2\pi R+\Omega Rt$, in K, while the point traveled ΩRt. Hence, the ratio of point speed to light speed is given by

$$\Omega R/c=\Omega Rt/(2\pi R+\Omega Rt),$$

or

$$t=2\pi R/(c-\Omega R). \quad (4)$$

If the system were at rest, $\Omega=0$, we would have $t=2\pi R/c$. Hence, for $\Omega\neq 0$, $\delta t=t-2\pi R/c$. Substitution of t from Eq. (4) gives

$$\delta t=2\pi\Omega R^2/[c(c-\Omega R)]\simeq 2\Omega A/c^2, \quad (5)$$

for $\Omega R\ll c$, with $A=\pi R^2$.

Suppose now that we think of having rest clocks set at intervals along the equator, and that we synchronize them by the following procedure. We say that if a light signal moves from clock 1 to clock 2, through a distance L, the time t_2 of reception of the signal by 2 must be related to the time t_1 of emission by 1 by the relation: $t_2-t_1=L/c$. We will say that the synchronization is made along the line of equatorial clocks in an eastward direction. Then, since the Earth also turns into the east, each time interval between clocks, L distance apart, will be slightly increased by the Sagnac effect. Since the increase is $2\Omega A/c^2$ for $L=2\pi R$, it will be

$$(L/2\pi R)(2\Omega A/c^2)=LR\Omega/c^2$$

over a distance L. That is, compared with clocks in the hypothetical non-rotating system S, the synchronization to the east will give a slow time: An earth clock will be set at $t=L/c$ a bit later ($\delta t=LR\Omega/c^2$) than would be an S clock at the same point.

Now let us look at the term $(-vR\Omega/c^2)\Delta t_0$, $\lambda=0$, Eq. (2), that is responsible for the east–west asymmetry. Over a distance L, $\Delta t_0=L/v$, and hence the time loss over that distance is also $LR\Omega/c^2$. Since L is arbitrary, we see that the "Sagnac loss" in the clock synchronization exactly compensates the time loss prescribed as resulting from the Earth's rotation in Eq. (2). If one were moving to the west, the result of the Sagnac effect in the system of equatorial to-the-west synchronized clocks would be to give a time gain, relative to S clocks, of $LR\Omega/c^2$ over a distance L. Likewise, the westward flying clock gains that same time increment. We can conclude that *in the system* of appropriately synchronized equatorial clocks there would be no observed difference in time rate between clocks being flown eastward and those being flown westward. [There would be a difference between the clocks when they returned to their starting point, but also (see Sec. IV) there would be a comparable discontinuity in the system of synchronized clocks.] We would, of course, still have the $(-v^2/2c^2)\Delta t_0$ time loss, Eq. (2), for either clock, but this is just what is to be expected from $\Delta t=\Delta t_0/\gamma$, $1/\gamma\simeq 1-v^2/2c^2$, as predicted from the energy transformation. We restore consistency by utilization of a theoretically (and empirically) established correction, that is, the Sagnac effect, in the synchronization of the reference clock system.

IV. THE HAFELE–KEATING DISCONTINUITY

We consider again a synchronized set of equatorial clocks. If we go around eastward there will have been a total "Sagnac" clock loss of $2\pi R^2\Omega/c^2=2A\Omega/c^2$ when we return to the starting point, or a similar gain if on returning to the starting point we have moved to the west. That is, there must be a discontinuity in the clock system at the starting point, such that we gain or lose $\delta t=2A\Omega/c^2$ as we go across the starting point toward the east or west, respectively. Hence, if our earth clocks are synchronized by electromagnetic wave propagation, we must have a discontinuity somewhere along each line of latitude if we are to have a single-valued time system. We might call this, I suggest,[7] the Hafele–Keating discontinuity; its value at latitude λ would be $(2A\Omega\cos\lambda)/c^2$, so the magnitude of the jump would decrease as one went from equator to either pole. The effect is, to be sure, a very small one: $2A\Omega/c^2\simeq 200$ nsec for the Earth. However, it might be of significance in a terrestrial time mesh of high precision.

It could be avoided if all terrestrial rest clocks were based on meridian transit of a star, giving, in effect, an S-clock synchronization. Or, equivalently, theoretical correction for the Sagnac effect could be made in signal synchronization. With an "S-clock" synchronization there would still be a $-\Omega^2R^2/2c^2$ relativistic rate factor for rest earth clocks relative to the hypothetical S clocks. It might be a disadvantage that in such a clock system the Hafele east–west asymmetry would be manifest for moving clocks, but there would be a compensating gain in that we would not have two systems of clock readings which because of the Sagnac effect are different depending on whether synchronization was to the east or to the west.

There is, however, a further remarkable feature of the east–west clock effect. Returning again to the term $(-vR\Omega/c^2)\Delta t_0$ of Eq. (2), we see that $L=2\pi R$ gives $\Delta t_0=2\pi R/v$ and the term becomes $-2\pi R^2\Omega/c^2=-2A\Omega/c^2$. That is, the relativistic time loss prescribed by the term is *independent* of the speed at which the clock is moved. Hafele and Keating confirmed the loss (and gain, for westward flight) for their jet-flown atomic clocks. But our results show us that we can expect atomic clocks carried at any speed around the world will give the same effect: a loss of $(2A\Omega\cos\lambda)/c^2$ if carried eastward at latitude λ, a gain if carried westward.

Since the Hafele–Keating discontinuity, even though relativistic, is independent of speed, we can think of it as similar in that respect to the time jump required by the International Date Line. That discontinuity results, of course, because the traveller on completion of a circumpolar journey around the world, at any speed, has made one more (or less) axial rotation than has the non-traveller. So, in order to have a single-valued time measure we set up the Date Line. Likewise, the traveller carrying an atomic clock would either have to make periodic adjustments as he went around the world, or make a single increase (decrease) on completion of a circuit. But as already noted, there is the difference of a $\cos\lambda$ dependence for the clock effect. Any actual atomic clock would presumably be carried around the world on a path of varying latitude and integration would have to be made over the various λ values (as was done by Hafele and Keating[1]).

V. THE UNIVERSAL CLOCK-RATE TRANSFORMATION

Since the gh/c^2 term in Eq. (2) is comparable in magnitude to the other two terms for the conditions of the Hafele–Keating observations, we know that their positive results give a confirmation of the general relativistic time-rate change for the macroscopic atomic clock. Can we also conclude, considering the other two terms as special relativistic effects, that the experiment confirms a $\Delta t=\Delta t_0(1-w^2/c^2)^{1/2}$ time-rate change for any clock moving at relative uniform speed w? I shall now present an argument to the effect that the "kinetic" transformation on the clocks, given by the other two terms, is also in fact properly regarded as being a consequence of the general rather than the special relativity theory.[9]

The around-the-world paths of the clocks in the Hafele–Keating experiment are, of course, circular. Also, the photons, or electromagnetic waves, in the cesium atomic clock are themselves subject to general relativistic frequency changes and the clock rate follows these changes. Thus, the alteration of photon frequency with height in a gravitational field is manifested in the observed time increase (the gh/c^2 term) that resulted from carrying the atomic-clock photons to a higher gravitational potential. Likewise, by the Equivalence principle, the accelerated, circular motion of the clocks in system S should give a gravitational potential difference that entails a photon frequency decrease. A gravitational potential difference $\Delta\phi$, we recall, gives rise to a clock rate change $\delta t=(\Delta\phi/c^2)t$, the faster rate being at the higher potential.[10]

Specifically, let us say the atomic clock has a speed $w=\omega r$, where w is $\Omega R\cos\lambda$, or $\Omega R\cos\lambda+v$, depending on whether we are speaking of the rest or flying-earth clock. In the earth system, the clock experiences a centrifugal force per unit mass of $F_r=w^2/r=\omega^2r$ in consequence of its motion and, by the equivalence principle, this force is equivalent to the action of a gravitational field. Since $F_r=-\partial\phi/\partial r$, the corresponding gravitational potential difference $\Delta\phi$ is $-\omega^2r^2/2$. But then the photons in the clock must experience a rate decrease such that

$$\delta t=(\Delta\phi/c^2)t=-(\omega^2r^2/2c^2)t=(-w^2/2c^2)t.$$

Or, for $t = \Delta t_0$,

$$\Delta t = \Delta t_0 + \delta t = \Delta t_0(1 - w^2/2c^2) \simeq \Delta t_0(1 - w^2/c^2)^{1/2},$$

which is the same as the special relativistic equation used in the derivation of Eq. (2). We can say, therefore, that the atomic clocks in the Hafele–Keating experiment show the observed motional time changes because of the effect of the accelerated circular motion on photon frequencies.

In the preceding derivation we have used for circular motion an equivalence between the special-relativistic kinetic time effect and the general-relativistic time effect to first order in v^2/c^2. Basically, since the motion is circular, it probably is best to consider the effect as a general-relativistic one. The question sometimes arises: Why do we not have both the kinetic (special relativistic) and the general-relativistic effects? The answer is that we do not when the general-relativistic effect is itself a consequence of the circular motion, by virtue of the equivalence principle. In contrast, when motion is in the presence of a permanent gravitational field we do have both as, indeed, in the Hafele–Keating experiment where there is both the gh/c^2 effect resulting from the Earth's gravitational field and the kinetic (circular motion) effects which may be calculated either from special or general-relativistic equations.

The question remains: Will any clock, as well as any other kind of physical process, undergo a time-rate decrease, $t' = t(1 - w^2/c^2)^{1/2}$, simply as a consequence of *uniform* relative motion at speed w? In view of the meson-decay time and other observations, the only reasonable answer would seem to be an affirmative one for physical processes in which the conjugate relativistic energy and time transformations are involved. However, it is possible, within the framework of standard special relativity theory, to accept an invariance (under relative uniform motion) of time rate for thermodynamic (macroscopic) clock processes which are not themselves in energy interaction with the observer.[11] This alternative, hypothetical interpretation involves a deviation from the customary extension of the Lorentz time transformation to all processes and, I judge, that few physicists would expect it to be correct. But we cannot, I suggest, consider the issue of universal time-rate change closed until a test has been made with uniform motion. Nonetheless, in showing a relativistic time transformation with circular motion for a literal macroscopic clock, Hafele and Keating have notably advanced our firm knowledge about what to many of us is one of the most important and interesting questions of modern physics.

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