Geometrical Appearances at Relativistic Speeds

G. D. SCOTT AND H. J. VAN DRIEL
Department of Physics, University of Toronto, Toronto 5, Canada
(Received 10 February 1970; revision received 2 April 1970)

Geometrical appearances at relativistic speeds ($\beta = 0.5$, 0.9, and 0.995) are illustrated for the following examples: (i) the celestial sphere with a number of constellations, (ii) the surface features of a sphere passing close to an observer, and (iii) a train of rectangular boxes. The figures make clear the nature of the distortions which occur in appearances, indicate the limited significance of the so-called "apparent rotation," and show the conditions under which the Lorentz contraction can be seen or photographed. Though a sphere remains essentially circular in outline, the apparent cross section may be grossly distorted and under some conditions the outside surface of the sphere appears concave.

GEOMETRICAL APPEARANCES AT RELATIVISTIC SPEEDS

The visual appearance of objects moving at high speeds is a subject with many intriguing aspects. Since the papers of Penrose,[1] Terrel,[2] and Weiskopf[3] which sparked a new interest in the matter, there have been a number of publications on closely related topics[4–9] and recent elementary texts on relativity usually devote a section to the subject (e.g., French,[10] Resnick[11]). However, statements such as the following which might

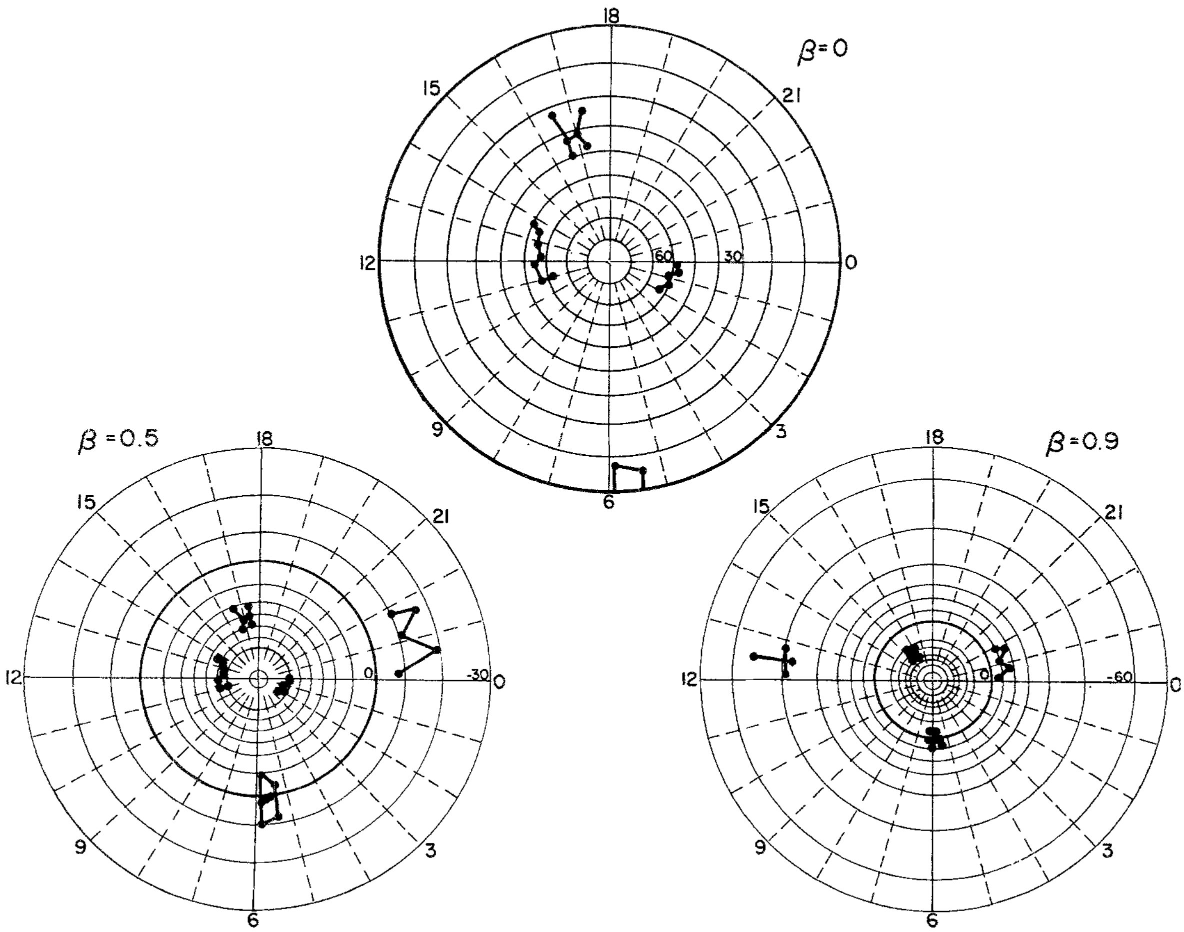

FIG. 1. Views of the northern celestial hemisphere as seen by an observer at the center and travelling towards the north celestial pole. For the view at rest ($\beta = 0$) the constellations shown are the Big Dipper, Cassiopeia, and Hercules. At the higher speeds Orion, part of Aquarius and the Southern Cross come into the field of view.

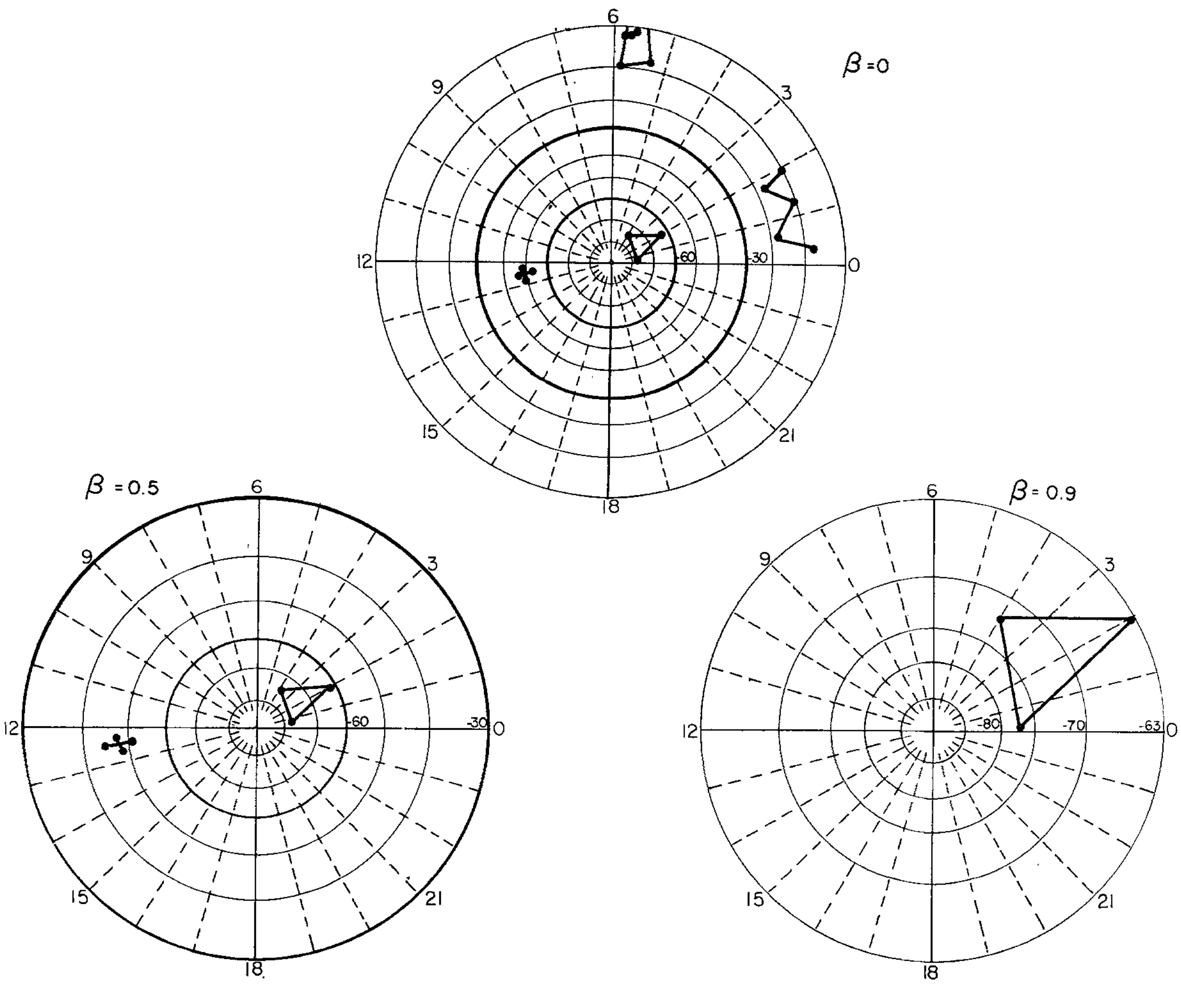

FIG. 2. Views of the southern celestial hemisphere as seen by an observer travelling towards the north celestial pole. The constellations indicated are part of Orion, part of Cetus, the Southern Cross, and Hydrus. Only Hydrus (shown as a triangle) remains at $\beta=0.995$.

seem to be based on the papers of Penrose and Terrel can be misleading: "A moving sphere always appears as a sphere"; "The Lorentz contraction cannot be photographed"; "An object in motion suffers an apparent rotation."

The purpose of this paper is to clarify the main features of the visual appearance of objects moving at relativistic speeds by illustrating a number of examples in some detail.

The various ways in which the geometry of a moving object may be determined have been discussed by McGill.[9] The visual appearance considered in the following examples is that which could be seen by an observer: It is a snapshot photograph of the object which is assumed to be self-luminous, or uniformly and continuously illuminated. The exposure time must be short enough to prevent blurring of the image, a condition that can always be realized in principle even at speeds close to that of light by imagining that the size and distance of the object are sufficiently large. Only the geometry of the image is discussed. The interesting effects of intensity and color change[3,8] (Doppler shifts) are not treated.

CALCULATION OF IMAGES

The appearances can all be calculated from the well-known expressions for the relativistic aberration, but it is generally simpler to use the concepts of time of flight and the Lorentz contraction. Consider two frames O and O' coincide at $t=t'=0$. If a plane has equation $x'=d$ in O', then at 0 to $t=$

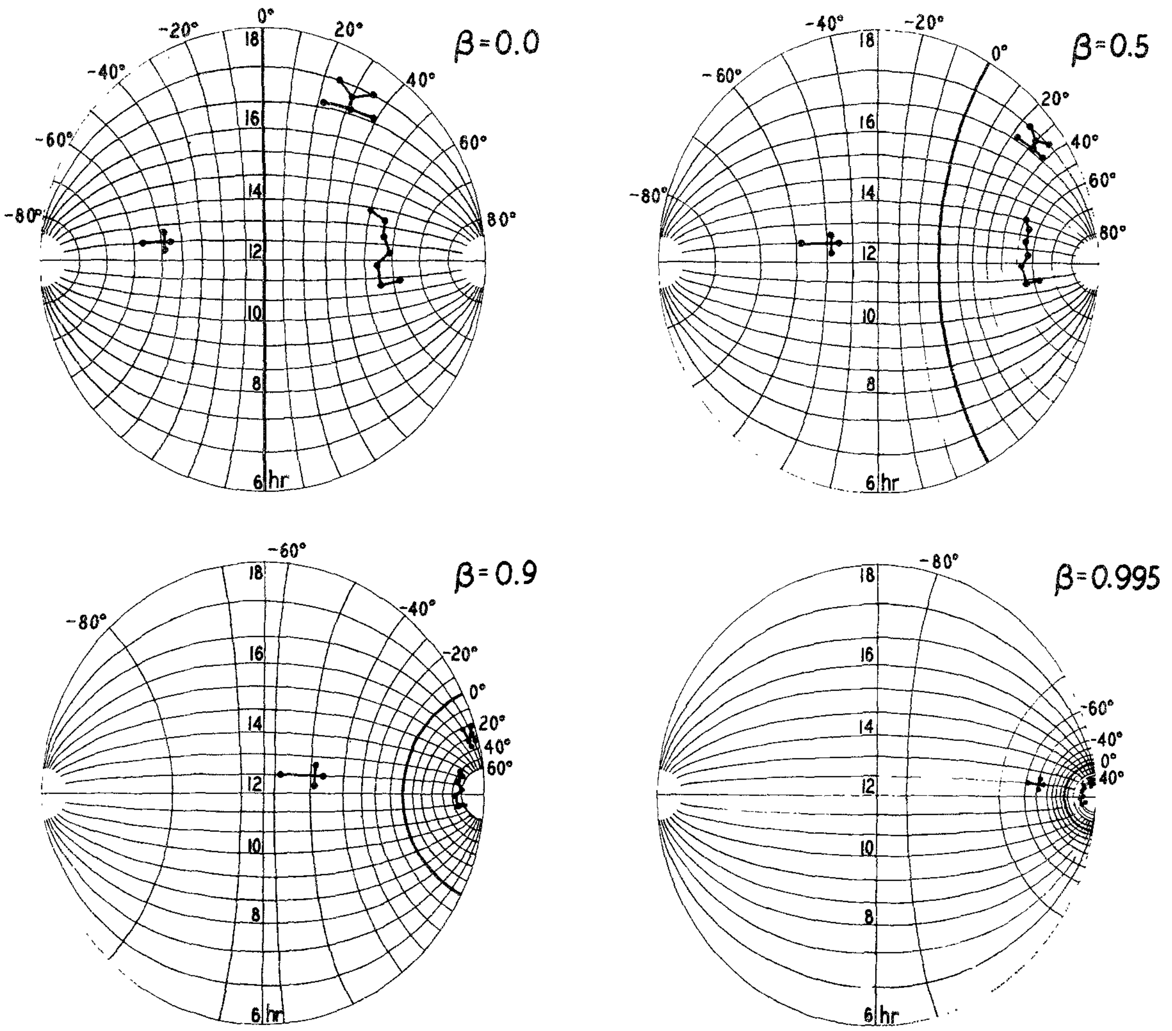

FIG. 3. Views of the western celestial hemisphere as seen by an observer travelling towards the north celestial pole. The constellations shown are the Big Dipper, Hercules, and the Southern Cross.

an observer at the origin of O, the plane will appear to have coordinates satisfying the equation

$$[(x-\gamma d)^2/\beta^2\gamma^2 d^2]-(y^2/d^2)-(z^2/d^2)=1.$$

Hence, any point with coordinate (x', y', z') in O' will, to the observer at the origin in O at $t=0$, appear to have coordinates

$$[\gamma x'-\beta\gamma(x'^2+y'^2+z'^2)^{1/2}, y', z'].^7$$

As usual, γ is $(1-\beta^2)^{-1/2}$.

In order to illustrate the image, the apparent angular position of points of the objects must be projected onto a plane. The projection is an essential ingredient in showing the appearance of an object. Any projection will generally introduce some distortion in appearance which is quite distinct from that associated with the relativistic distortion. Stereographic projection, such as is employed in crystallography, was used for all the cases of the spheres since it produces relatively small distortions for wide angles (up to 90°) from the direction of the "camera" axis. Central projection as in the pin hole camera is used for the "box cars" since straight lines in the object then become straight lines in the image, at least for the object at rest.

The detailed calculations were carried out by computer. Shirer and Bartell[12] have also developed a program for relativistic appearances.

THE CELESTIAL SPHERE

Points on the celestial sphere—in particular the various constellations—would be perhaps the most obvious objects to a high speed space traveller. The way in which angular positions on the celestial sphere will change with speed makes an excellent example of relativistic aberration.[3]

The observer is considered as moving to the north, and three views are illustrated in Figs. 1–3: to the north (ahead), to the south (behind), and to the west (to the left as the observer faces forward), respectively. In each case, the rest view and three different speeds $\beta = 0.5$, 0.9, and 0.995 are given. A few well-known constellations are shown as well as representative latitude and hour circles. The main characteristic of the appearance is the angular demagnification in the forward direction (to the north) which of course increases with speed and the corresponding angular magnification in the backward direction (to the south).

APPEARANCE OF A SPHERE

A spherical body is certainly another object which is to be viewed by a space traveller. Several authors[1,4] have shown that a luminous sphere will always present a circular disk to the observer regardless of its relative motion. The sphere will not in general give a circular image in a camera, but will do so only when the latter is directed at the sphere in such a way that the center of the circle is on the axis of the camera. That direction will not be towards the center of the sphere except at zero velocity. In general the sphere will produce a somewhat elliptical image, but it is the projection which produces the noncircular shape.

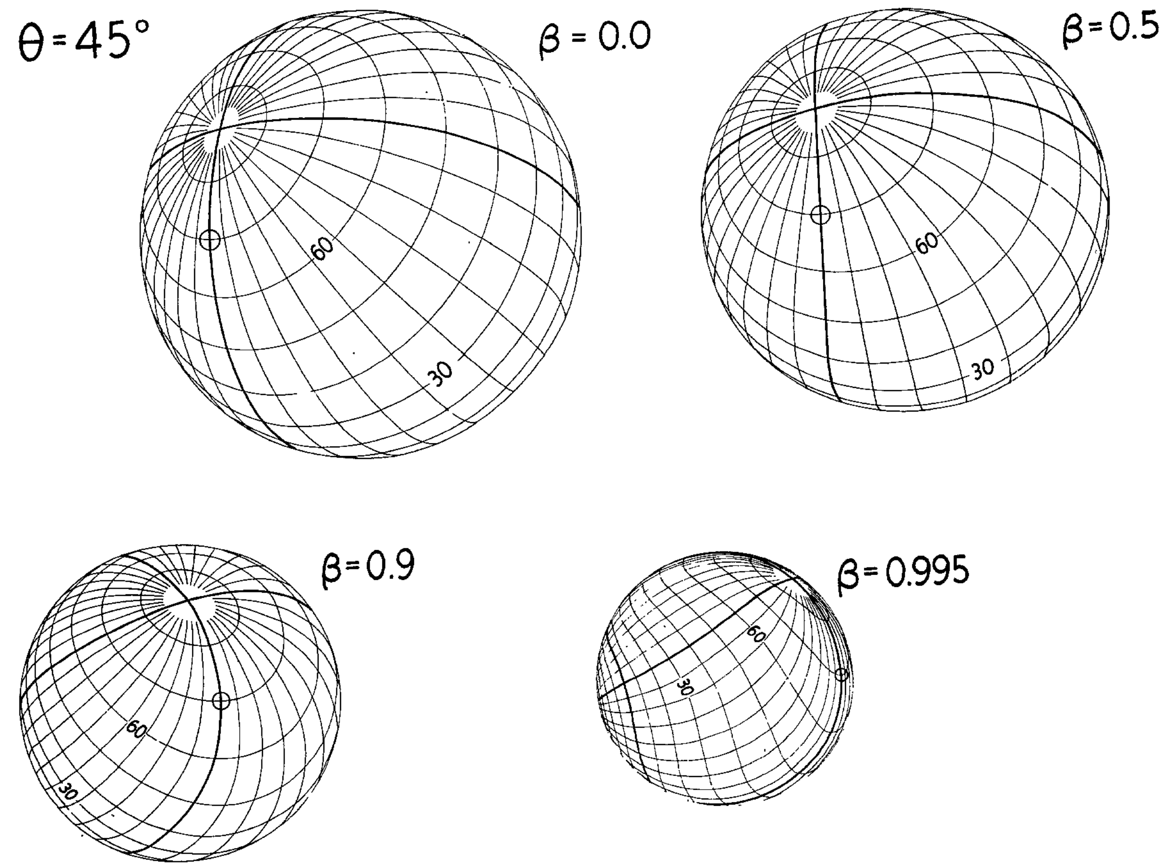

FIG. 4. Appearance of a sphere approaching an observer at various speeds β. The center of the sphere is moving along a line which is at a distance of one sphere diameter from the observer. As described in the text, the sphere is tilted by 70° to display the north pole. The direction from the point circled on the sphere to the observer makes an angle θ equal to 45° to the line of motion.

Figures 4–6 illustrate the appearance of a sphere in three different directions $\theta=45°$, $90°$, and $135°$, as it passes an observer with the line of the motion of its center at a distance of its diameter from the observer. Circles of latitude and longitude are shown on the sphere, which is tilted by 70° in a plane perpendicular to its line of motion so as to reveal the north pole and to indicate more clearly the surface distortion which occurs. The angle θ is the direction of the axis of the camera relative to the line of motion: The axis of the camera is in each case directed at the point 70° north latitude on the longitude circle which is at right angle to the line of motion. For each direction the view at three speeds is given, $\beta=0.5$, 0.9, and 0.995. Under all circumstances it is seen that the outline of the sphere is very close to a circle. Slight deviations from a circle result from the stereographic projection onto a plane. The magnification effects already noted on the celestial sphere are evident to a limited extent in the sizes of the sphere at the higher speeds for 45° and 135°.

Two features of the appearance of the moving sphere are (i) the increasing distortion of the latitude–longitude grid with speed and (ii) the apparent rotation corresponding to the effect discussed by Terrel—the backside of the approaching sphere comes into view.

Although the sphere is, under all conditions, essentially circular in outline, a most striking aspect of its appearance at high speeds is that the surface facing the observer becomes flattened and even concave. Apparent cross sections of the sphere in the plane containing the observer and center line of motion are shown in Figs. 7 and 8. The distance to points on the sphere's surface could be obtained in principle by an optical range finder or by stereophotography. A three-dimen-

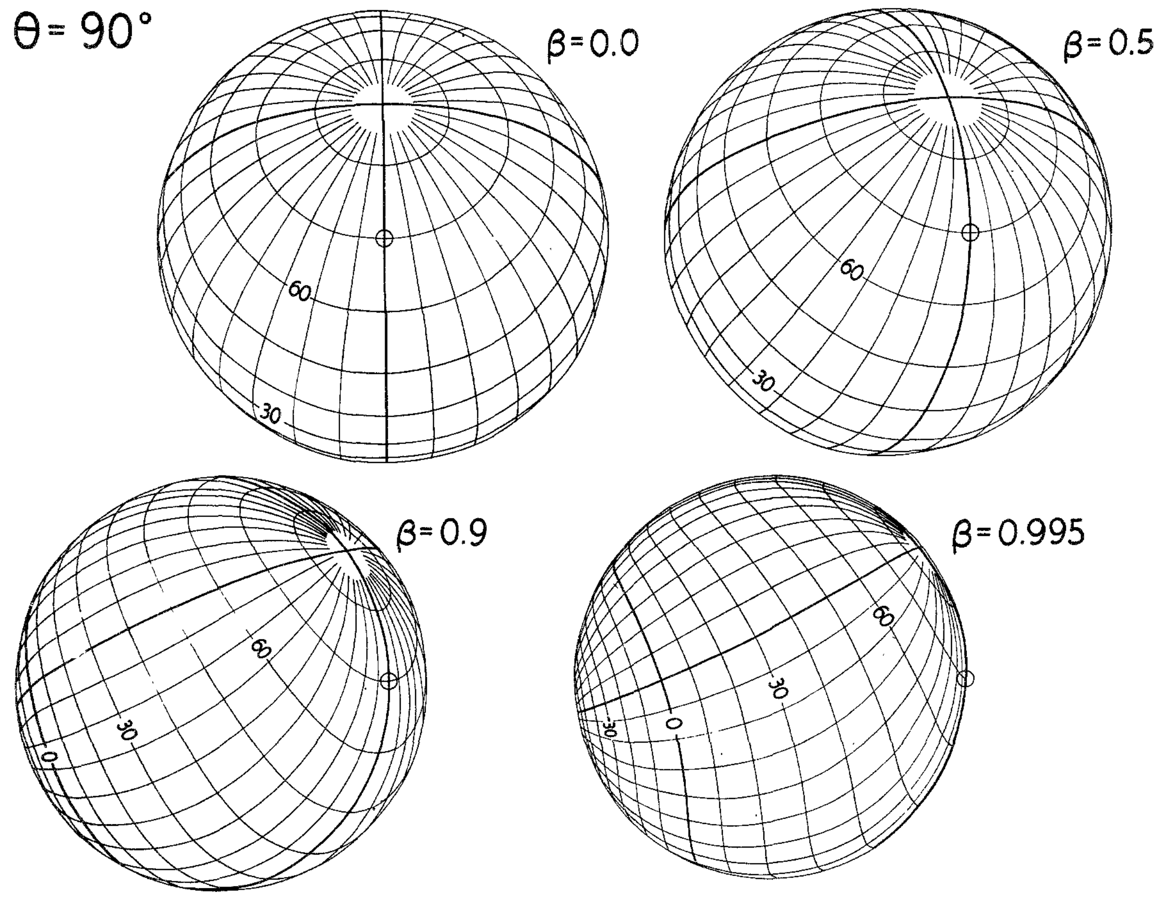

FIG. 5. Appearance of a sphere passing an observer. The conditions are as in Fig. 4 but the angle of observation θ is equal to 90°.

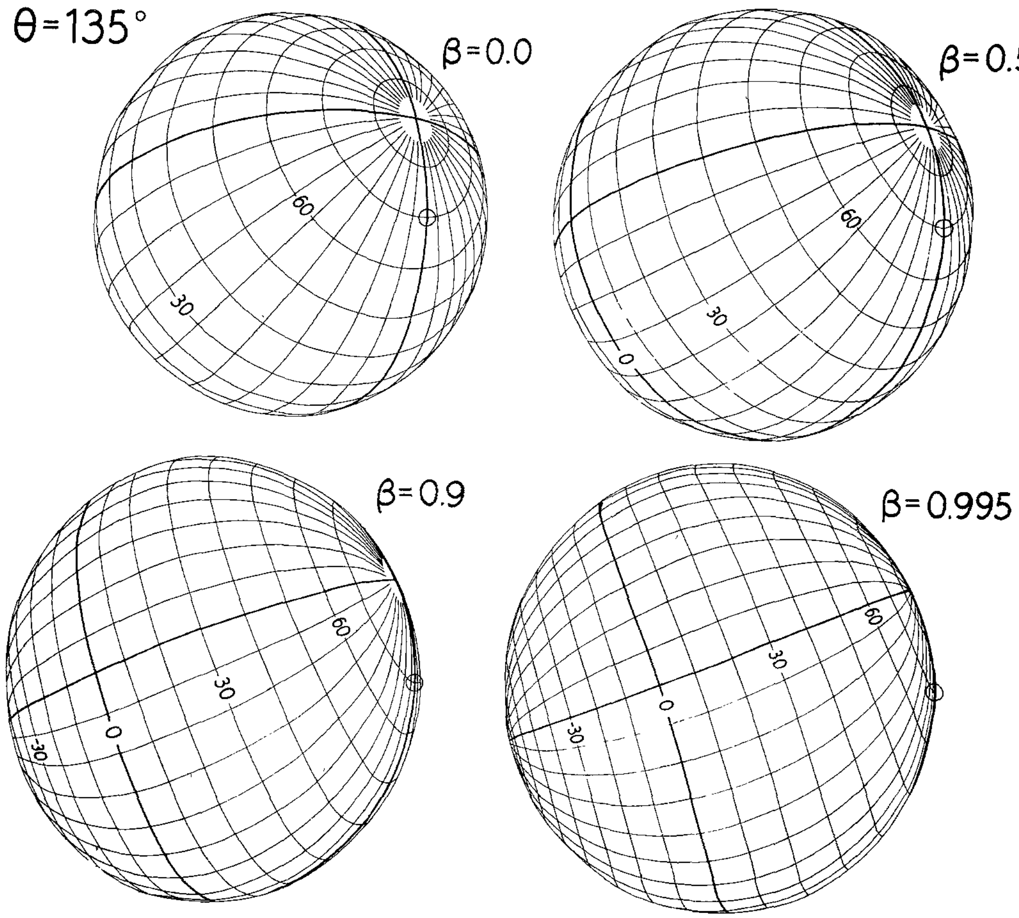

FIG. 6. Appearance of a sphere receding from an observer. Conditions are as in Fig. 4 except that the angle θ is equal to 135°.

sional stereoscopic (binocular) view of the sphere would reveal its flattened or concave surface, although as noted by McGill,[9] a steroscopic method is not an *ideal* way of obtaining the shape of a fast-moving object: The two images do not match exactly. The sphere could be considered transparent for determining the positions of points on the side away from the observer. The distortion in the cross section increases with β, but decreases as the distance from the observer increases. In Fig. 7, the distance from the observer to the line of motion of the sphere center is one diameter as in Figs. 4–6; in Fig. 8 it is ten diameters. For the more distant spheres the cross sections are almost ellipses with the major axes at an angle with respect to the line of motion which is about one-half the angle of observation θ: For example when $\theta=90°$ the ellipse is tilted at 45°.

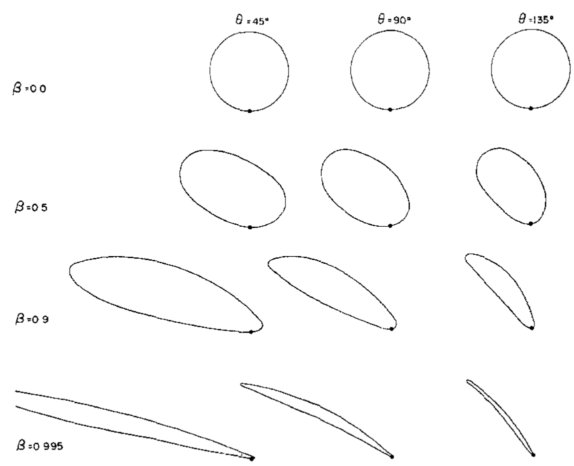

FIG. 7. Cross sections of the sphere images of Figs. 4–6, in the plane through the observer and the line of motion of the sphere center.

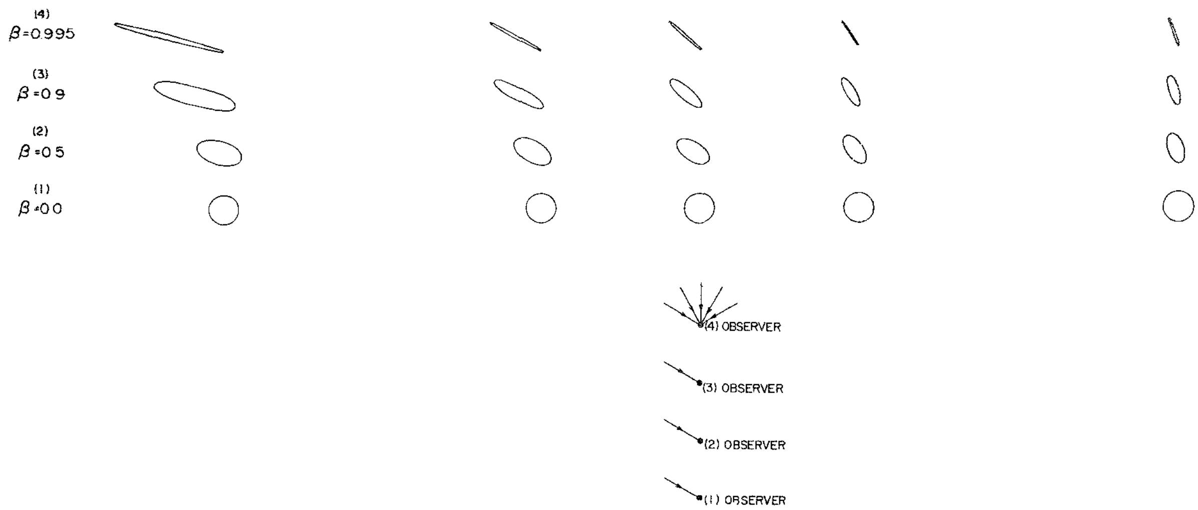

FIG. 8. Cross sections at various speeds of the images of passing spheres when the observer is ten sphere diameters from the line of motion of the center.

TRAIN OF BOX CARS

A third example is that of a linear group of rectangular prisms—they can be thought of as a train of boxcars on a straight track (Fig. 9). The observer is above the level of the train by an angle of 30°. Central projection has been used and the field shown is about ±45° from the camera axis. The views of the cars at rest and for three speeds $\beta=0.5$, 0.9, and 0.995 are illustrated. The increasing distortion in the appearance with increasing speed can readily be followed. The relation of the distortion to Terrel's "apparent rotation" can be appreciated but certainly an interpretation merely in terms of rotation is not possible. The Lorentz contraction can indeed be photographed but shows properly only in a direction perpendicular to the motion.

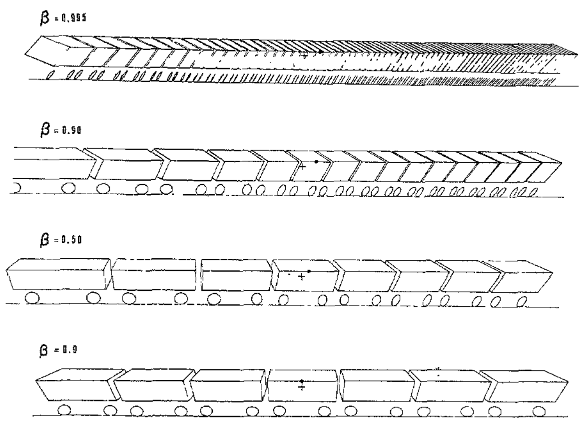

FIG. 9. Views of a train of boxcars (each 1×1×3 units) by an observer 10 units away at an angle 30° above the plane of the tracks. The axis of the observer's camera is directed at the center of the middle car shown by a cross (+). The dot (•) marks the center of the near edge of the middle car in each case.

ACKNOWLEDGMENT

The authors wish to acknowledge financial assistance from the National Research Council of Canada.

[1] R. Penrose, Proc. Cambridge Phil. Soc. **55**, 137 (1959).
[2] J. Terrel, Phys. Rev. **116**, 1041 (1959).
[3] V. F. Weiskopf, Phys. Today **13**, No. 9, 24 (1960).
[4] M. L. Boas, Amer. J. Phys. **29**, 283 (1961).
[5] H. A. Atwater, J. Opt. Soc. Amer. **52**, 184 (1962).
[6] S. Yngstrom, Arkiv Fysik **23**, 367 (1962).
[7] G. D. Scott and M. R. Viner, Amer. J. Phys. **33**, 534 (1965).
[8] S. Moskowitz, Sky and Telescope **33**, 290 (1967).
[9] N. C. McGill, Contemp. Phys. **9**, 33 (1968).
[10] A. P. French, *Special Relativity* (Norton, New York, 1968), p. 149.
[11] R. Resnick, *Introduction to Special Relativity* (Wiley, New York, 1968), p. 77.
[12] D. L. Shirer and T. W. Bartel, Amer. J. Phys. **35**, 434 (1967).