in the form

$$\int_{t_1'}^{t_2'} \mathbf{F}'dt' = m(\mathbf{u}_2'+\mathbf{v}) - m(\mathbf{u}_1'+\mathbf{v}) = m\mathbf{u}_2' - m\mathbf{u}_1'. \quad (7)$$

Therefore, O' will write the momentum equation in the same form O does. In particular, if P moves under no forces in the S frame, it will also have a forceless motion in S', and (6) and (7) show that in this case linear momentum is conserved in both frames.

Next, O will define the particle's kinetic energy at time t as $\frac{1}{2}m\mathbf{u}^2$, while O' will define the KE to be $\frac{1}{2}m\mathbf{u}'^2$ at the same instant (t'). According to O, the energy equation for P is

$$\int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F}\cdot d\mathbf{r} = \tfrac{1}{2}m\mathbf{u}_2^2 - \tfrac{1}{2}m\mathbf{u}_1^2. \quad (8)$$

Using (2) in differential form, and remembering that $\mathbf{F}=\mathbf{F}'$, we may write the left side of (8) as

$$\int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F}\cdot d\mathbf{r} = \int_{\mathbf{r}_1',t_1}^{\mathbf{r}_2',t_2} \mathbf{F}\cdot (d\mathbf{r}'+\mathbf{v}dt)$$
$$= \int_{\mathbf{r}_1'}^{\mathbf{r}_2'} \mathbf{F}'\cdot d\mathbf{r}' + \mathbf{v}\cdot \int_{t_1}^{t_2} \mathbf{F}dt$$
$$= \int_{\mathbf{r}_1'}^{\mathbf{r}_2'} \mathbf{F}'\cdot d\mathbf{r}' + \mathbf{v}\cdot (m\mathbf{u}_2 - m\mathbf{u}_1)$$

upon using (6). Rearranging, with the use of (8), we get

$$\int_{\mathbf{r}_1'}^{\mathbf{r}_2'} \mathbf{F}'\cdot d\mathbf{r}' = \tfrac{1}{2}m\mathbf{u}_2^2 - \tfrac{1}{2}m\mathbf{u}_1^2 - m\mathbf{u}\cdot\mathbf{v} + m\mathbf{u}_1\cdot\mathbf{v}$$
$$= \tfrac{1}{2}m(\mathbf{u}_2-\mathbf{v})^2 - \tfrac{1}{2}m(\mathbf{u}_1-\mathbf{v})^2, \quad (9)$$

showing that the energy principle has the same form in S' as it does in S since by (3) $\mathbf{u}_2'=\mathbf{u}_2-\mathbf{v}$, $\mathbf{u}_1'=\mathbf{u}_1-\mathbf{v}$.

Finally, we demonstrate the form invariance of the principle of angular momentum for a single particle. At time $t(=t')$, the angular momentum will be defined by O, O' to be, respectively,

$$\mathbf{h}=\mathbf{r}\times m\mathbf{u}, \qquad \mathbf{h}'=\mathbf{r}'\times m\mathbf{u}'. \quad (10)$$

Differentiating (10), we get

$$d\mathbf{h}/dt = (d\mathbf{r}/dt)\times m\mathbf{u} + \mathbf{r}\times m(d\mathbf{u}/dt) = \mathbf{r}\times\mathbf{F}, \quad (11)$$

$$d\mathbf{h}'/dt' = (d\mathbf{r}'/dt')\times m\mathbf{u}' + \mathbf{r}'\times m(d\mathbf{u}'/dt')$$
$$= \mathbf{r}'\times\mathbf{F}', \quad (12)$$

where we have used (4) and (5). Equations (11) and (12) constitute the principle of angular momentum in the unprimed and primed frames, respectively. If the applied force $\mathbf{F}$ is zero in S it will be in S', and then (11) and (12) give the principle of conservation of angular momentum, valid in both frames. This may also be seen as follows:

$$d\mathbf{h}'/dt' = \mathbf{r}'\times\mathbf{F} = (\mathbf{r}-t\mathbf{v})\times\mathbf{F} = \mathbf{r}\times\mathbf{F} - t\mathbf{v}\times\mathbf{F}$$
$$= d\mathbf{h}/dt - t\mathbf{v}\times\mathbf{F},$$

using (12), (11), and (2), remembering that $\mathbf{F}'=\mathbf{F}$. Therefore, if $\mathbf{F}=0$ we have $d\mathbf{h}'/dt'=d\mathbf{h}/dt$.

In conclusion, the writer believes that the discussion presented above from the point of view of an observer in either of the inertial frames will help to emphasize the relativistic nature of the principles of energy and momentum in Newtonian mechanics, and to underline the fact that no inertial observer is privileged, a fact that is often not grasped by students of elementary mechanics.

* At present on leave from the Department of Mathematics, Massey University, Palmerston North, New Zealand.

Erratum: "Geometrical Appearances at Relativistic Speeds," G. D. Scott and H. J. van Driel [Amer. J. Phys. **38**, 971 (1970)]. The last line of the second column of p. 972 should read: "If a plane has equation $x'=d$ in 0′, then at $t=0$ to."